Lecture 09

Machine Learning 3:

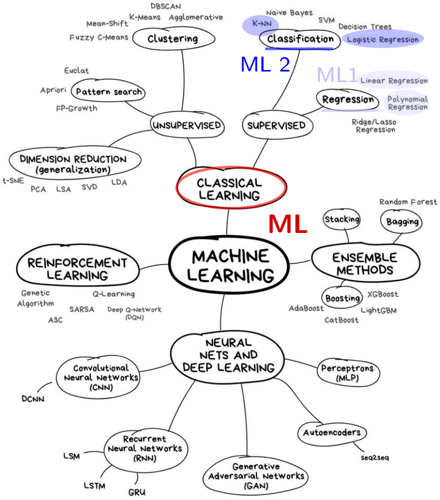
classification (part 2)

2024-10-16

Sébastien Valade

Introduction

Last week: classification *part-1* (ML2)



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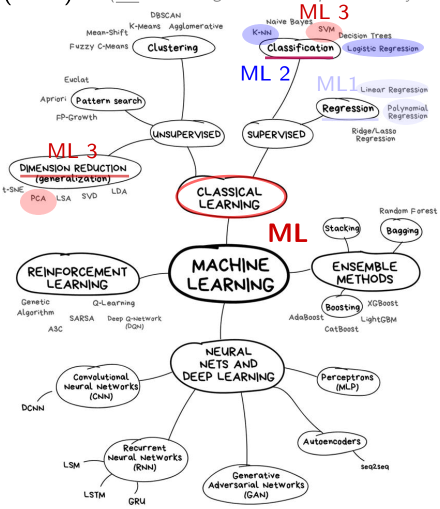
Introduction

Last week: classification *part-1* (ML2)
This week: classification *part-2* (ML3)

(NB: several figures were adapted courtesy of A. Ley & R. Hänsch from TU-Berlin)



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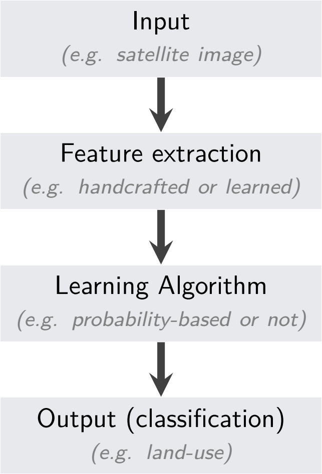
Introduction

Classification task

- Goal:
  Learn the mapping between low level **features**, and **high level information** *(e.g. semantic classes)*
- Steps:
  1. features extraction *(e.g. handcrafted | learned)*
  2. learning algorithm *(e.g. probablity-based | not)*
- Strategies:
  ⇒ last week:
  handcrafted features + probability-based learning
  ⇒ this week:
  learned features (PCA) + SVM learning



Input
(e.g. satellite image)

Feature extraction
(e.g. handcrafted or learned)

Learning Algorithm
(e.g. probability-based or not)

Output (classification)
(e.g. land-use)

1. Introduction
2. Principal Component Analysis (PCA)
    1. introduction
    2. how it works
    3. implementation steps
3. Classification algorithms (perceptron + SVM)
4. Exercise

Principal Component Analysis (PCA)

⇒ PCA is an **unsupervised learning** technique

- → in contrast to *supervised learning*, *unsupervised learning* algorithms operate on unlabeled data (we only have a set of k features $X_1, X_2, ..., X_k$ measured on n observations, without any associated target variable Y, thus we are not interested in any prediction task)

⇒ PCA is the most popular **dimensionality reduction algorithm**:

- → it is used to reduce data dimensionality, while preserving as much of the variance as possible
- → it is often used as data pre-processing technique before supervised techniques are applied (e.g. feature extraction to reduce computational load of the classifier)

⇒ Intuitive explanation: which angle captures the most information about the teapot?

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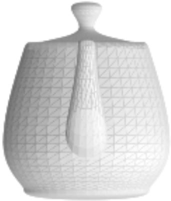

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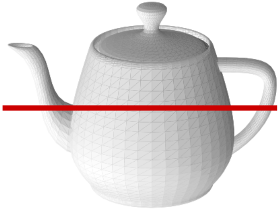

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= 1st "eigen vector"
(longest axis)

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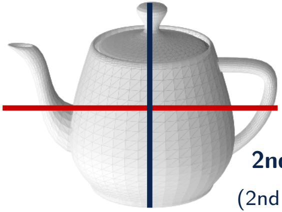
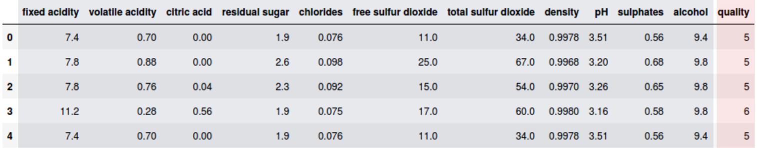


1st principal component
= 1st "eigen vector"
(longest axis)

2nd principal component
= 2nd "eigen vector"
(2nd longest axis ⊥ to 1st axis)

PCA toy example

*We have several wine bottles in our cellar, 11 **features** (alcohol, acidity, etc.) describe its **quality**. Which features best define it, are there related features (i.e. covariant) which are redundent?*



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0	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4	5
1	7.8	0.88	0.00	2.6	0.098	25.0	67.0	0.9968	3.20	0.68	9.8	5
2	7.8	0.76	0.04	2.3	0.092	15.0	54.0	0.9970	3.26	0.65	9.8	5
3	11.2	0.28	0.56	1.9	0.075	17.0	60.0	0.9980	3.16	0.58	9.8	6
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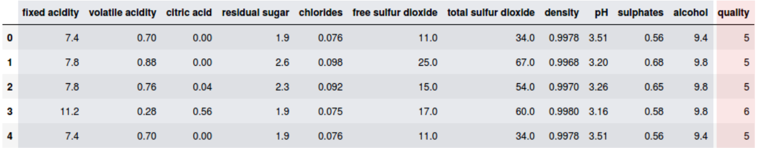

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⇒ **reduce data dimensions**

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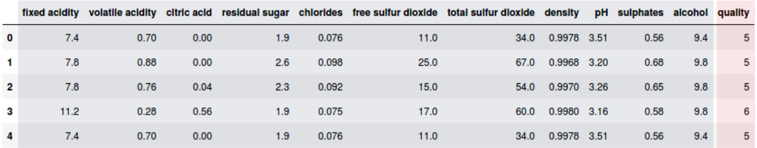

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instead it **defines new features** (using linear combinations of available features)
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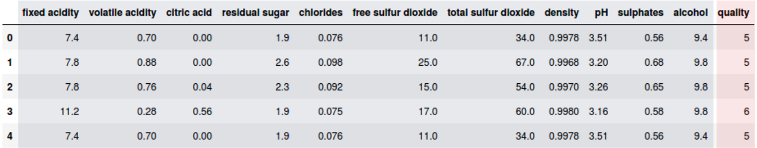

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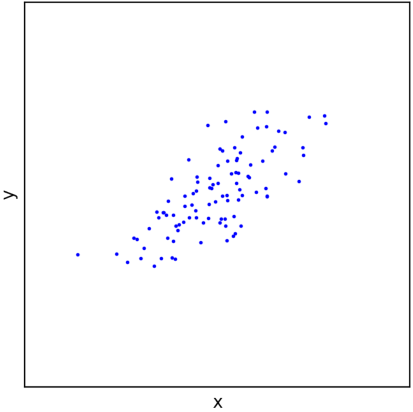
How?

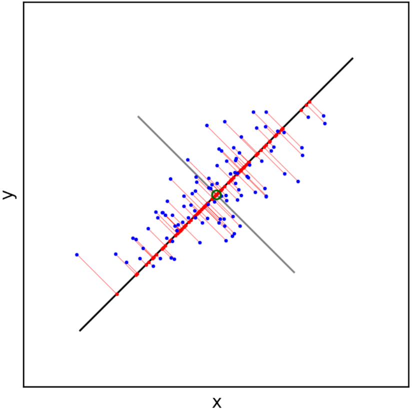
2.2. how it works

Consider 2 correlated features x and y:



Consider 2 correlated features x and y:

⇒ a **new "feature"** (red dots ●) can be constructed by drawing a line through the cloud and projecting all points onto it



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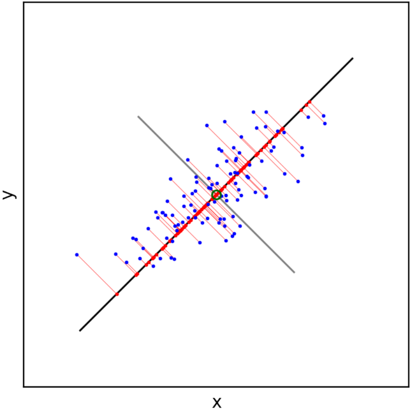

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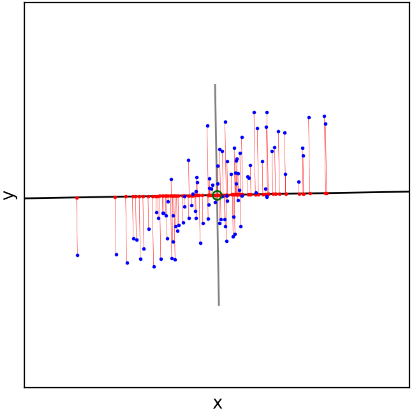
⇒ PCA will find the "best" line according to 2 criteria:

- maximum **variance** of the red dots (i.e., spread along black line)
- minimum **distance** to black line (i.e., length of red lines)



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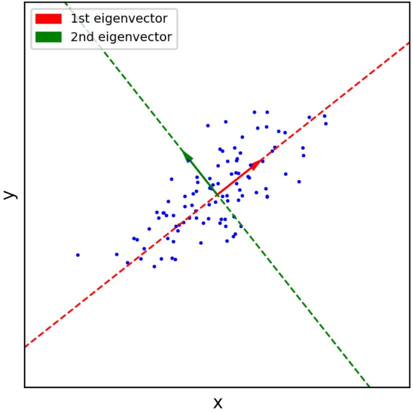

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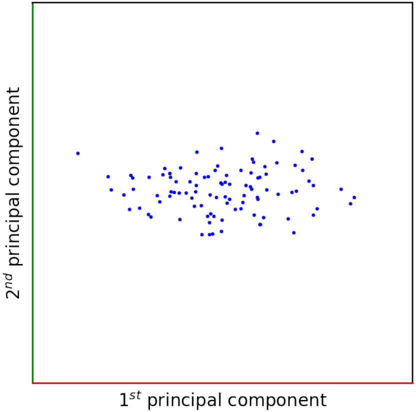
⇒ we can project the data on the principal components, and thereby reduce dimensionality

NB: if only one eigenvector was kept, the transformed data would have only one dimension



Implementation steps

Math reminders

$$\begin{aligned}\textbf{variance } \sigma^2 &= \text{measure of the ``spread'' or ``extent'' of the data about some particular axis} \\ &= \text{average of the squared differences from the mean} \\ &= \text{square of standard deviation } (\sigma) \\ var_x &= \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N} \\ var_y &= \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N}\end{aligned}$$

$$\begin{aligned}\textbf{covariance} &= \text{measure the level to which two variables vary together} \\ cov_{x,y} &= \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{N - 1} \\ \textbf{covariance matrix} &= \begin{bmatrix} var_x & cov_{x,y} \\ cov_{y,x} & var_y \end{bmatrix}\end{aligned}$$

Math reminders (continued)

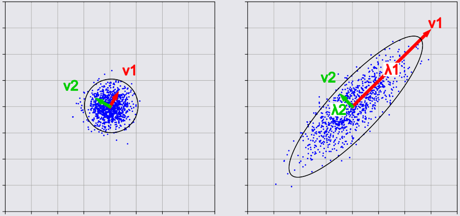
Covariance matrix $= \begin{bmatrix} var_x & cov_{x,y} \\ cov_{y,x} & var_y \end{bmatrix}$

Eigenvalue analysis of covariance matrix $\Rightarrow$ find directions with maximal variance

- **eigenvectors** $(\vec{v}_1, \vec{v}_2)$: represent the directions of the largest variance of the data
- **eigenvalues** (λ_1, λ_2): represent the magnitude of this variance in those directions

Determinant and trace of covariance matrix

- **determinant** $det(covmat) = \lambda_1 \lambda_2$: measures the "spread" of the data captured by the covariance matrix
- **trace** $trace(covmat) = \lambda_1 + \lambda_2$: measures the "total variance" captured by the covariance matrix



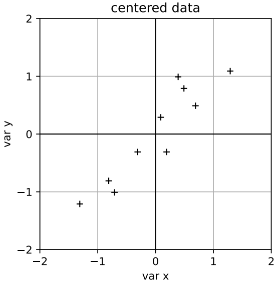
$covmat = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

uncorrelated variables

$covmat = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

highly correlated variables

Implementation steps (example with 2 variables)



```
x = x - x.mean()
y = y - y.mean()
```

1. center points around origin $(0, 0)$
2. compute **covariance matrix** → get **eigenvalues** & **eigenvectors** (= Principal Components) → sort by eigenvalue
   - ⇒ eigenvectors represent the directions of the largest variance of the data, eigenvalues represent the magnitude of this variance in those directions
   - ⇒ highest eigenvalue = direction with most variance (data dispersion) = 1^{st} principal component
3. project the data onto the **principal components** (PCs)
   - ⇒ if only 1 eigenvector was kept, the 2 original features (var x, var y) could be reduced to 1 dimension

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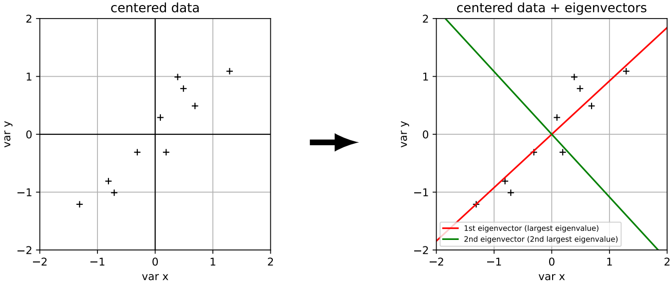

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```
cov = np.cov(x, y)
eig_val, eig_vec = np.linalg.eig(cov)
idx = np.argsort(-eig_val)
eig_val, eig_vec = eig_val[idx], eig_vec[:,idx]
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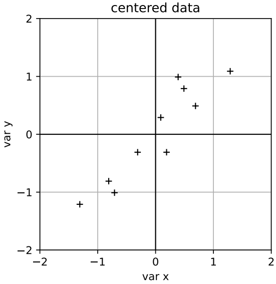

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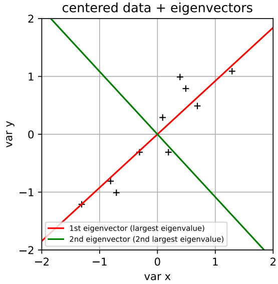

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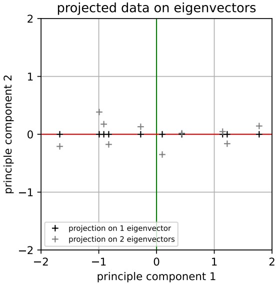

```
XY = np.array([x, y]).T
XY_proj = np.dot(XY, eig_vec[:2].T)
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2.3. implementation steps

Implementation steps (back to our toy-example on wine quality)

⇒ do the same with the 11 features: search for the principal components in a 11-dimensional space

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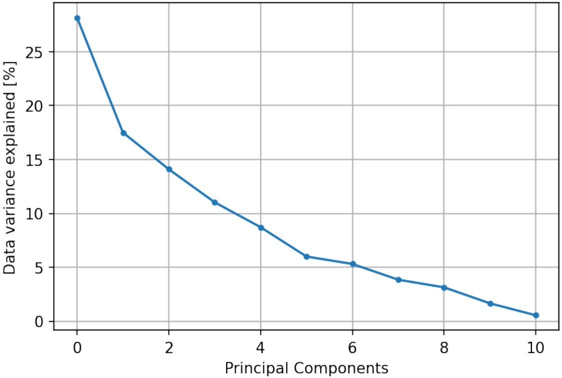

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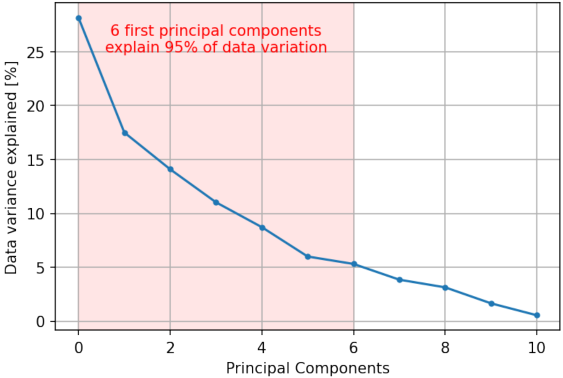

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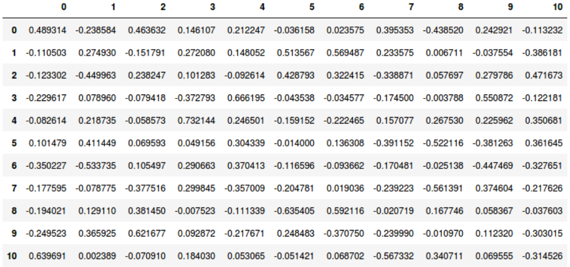
Q2: How do the 11 eigenvectors (PCs) relate to the original feature space?



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0	0.489314	-0.238584	0.463632	0.146107	0.212247	-0.036158	0.023575	0.395353	-0.438520	0.242921	-0.113232
1	-0.110503	0.274930	-0.151791	0.272080	0.148052	0.513567	0.569487	0.233575	0.006711	-0.037554	-0.386181
2	-0.123302	-0.449963	0.238247	0.101283	-0.092614	0.428793	0.322415	-0.338871	0.057697	0.279786	0.471673
3	-0.229617	0.078960	-0.079418	-0.372793	0.666195	-0.043538	-0.034577	-0.174500	-0.003788	0.550872	-0.122181
4	-0.082614	0.218735	-0.058573	0.732144	0.246501	-0.159152	-0.222465	0.157077	0.267530	0.225962	0.350681
5	0.101479	0.411449	0.069593	0.049156	0.304339	-0.014000	0.136308	-0.391152	-0.522116	-0.381263	0.361645
6	-0.350227	-0.533735	0.105497	0.290663	0.370413	-0.116596	-0.093662	-0.170481	-0.025138	-0.447469	-0.327651
7	-0.177595	-0.078775	-0.377516	0.299845	-0.357009	-0.204781	0.019036	-0.239223	-0.561391	0.374604	-0.217626
8	-0.194021	0.129110	0.381450	-0.007523	-0.111339	-0.635405	0.592116	-0.020719	0.167746	0.058367	-0.037603
9	-0.249523	0.365925	0.621677	0.092872	-0.217671	0.248483	-0.370750	-0.239990	-0.010970	0.112320	-0.303015
10	0.639691	0.002389	-0.070910	0.184030	0.053065	-0.051421	0.068702	-0.567332	0.340711	0.069555	-0.314526

Implementation steps (back to our toy-example on wine quality)

⇒ do the same with the 11 features: search for the principal components in a 11-dimensional space

NB: the maximum number of components is restricted by the number of features

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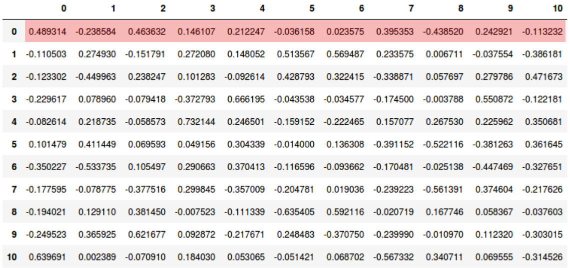

	0	1	2	3	4	5	6	7	8	9	10
0	0.489314	-0.238584	0.463632	0.146107	0.212247	-0.036158	0.023575	0.395353	-0.438520	0.242921	-0.113232
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Principal Component 1

PC 1 *= 0.49*feature0 + -0.24*feature1 + 0.46*feature2 + 0.15*feature3 + 0.21*feature4 + -0.04*feature5 + 0.02*feature6 + 0.40*feature7 + -0.44*feature8 + 0.24*feature9 + -0.11*feature10*

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Prediction accuracy of wine quality (categorical variable ⇒ classification task using kNN):

- using 11 original features ⇒ accuracy $= 0.79$
- using 6 first principal components ⇒ accuracy $= 0.78$

2.3. implementation steps

Implementation steps (back to our toy-example on wine quality)

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⇒ how about using PCA on images?
→ Sentinel-2 exercise: a crop of 900 pixels (4-bands 15×15 pixels) can be reduced fairly accurately to 32 points! (i.e., projection in a 32-dimensional space, first 32 pcs)

1. Introduction
2. Principal Component Analysis (PCA)
3. Classification algorithms (perceptron + SVM)
    1. Perceptron
    2. Support Vector Machine (SVM)
4. Exercise

Classification algorithms (perceptron + SVM)

Once features have been extracted, we can feed to the classifier! (recall last week lecture)

⇒ the classification algorithm needs to learn the **decision boundary** *(i.e. surface separating the different classes)* in an N-dimensional **feature space**:

- *probabilistic approaches*:
  - **Logistic Regression** ⇒ *last week*
  - **Softmax Regression** ⇒ *last week*
  - **Naive Bayes**
- *non-probabilistic approaches*:
  - **k-Nearest Neighbors (kNN)** ⇒ *last week*
  - **Perceptron** ⇒ today!
    → algorithm finding a hyperplane to separate classes, adjusting weights based on misclassified points
  - **Support Vector Machines (SVM)** ⇒ today!
    → algorithm finding the optimal hyperplane that maximizes the margin between classes
  - **Random Forest** ⇒ *next lectures*
  - **Convolutional Neural Networks (CNNs)** ⇒ *next lectures*

Classification algorithms (perceptron + SVM)

Choosing a classifier

- **Logistic regression (& Softmax)** *(last week lecture)*

  ⇒ probability-based linear classification method
  ⇒ *advantage*: simple, fast, interpretable
  ⇒ *disadvantage*: limited to linear decision boundaries

- **k-Nearest Neighbor (kNN)** *(last week lecture)*

  ⇒ label images by comparing them to (annotated) images from the training set
  ⇒ *advantage*: non-linear decision boundaries
  ⇒ *disadvantage*: classifier needs to keep all training data for future comparisons with the test data *(classifying test images is expensive as it requires comparison to all training images, inefficient with v. large datasets* $\geq$*GB)*

- **Support Vector Machines** *(this week lecture)*

  ⇒ parametric linear classification method
  ⇒ *advantage*: once the parameters are learnt, training data can be discarded *(classification of new images is fast: simple matrix multiplication with learned weights, not an exhaustive comparison to every single training data)*

- **Convolutional Neural Networks** *(coming weeks)*

  ⇒ CNNs map image pixels to classes, but the mapping is more complex and will contain more parameters
  ⇒ *advantage*: very powerful
  ⇒ *disadvantage*: needs LOTS of data!

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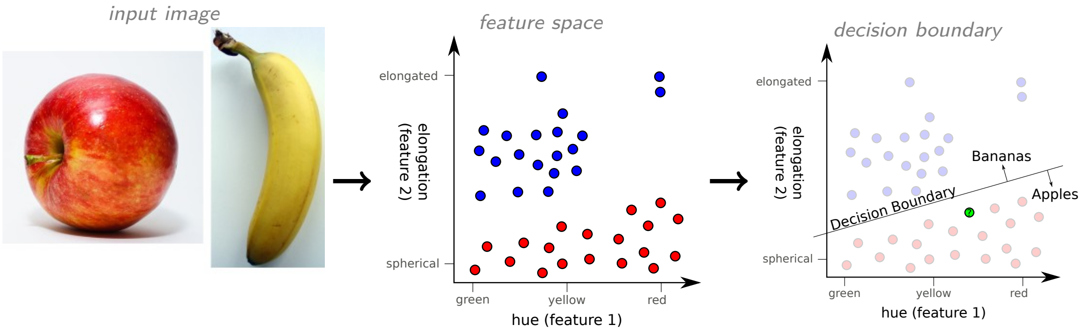
Recall our toy example from last week: *classify fruit images into either bananas or apples*



⇒ *how is the decision boundary learned?*

Perceptron classifier

⇒ algorithm which classifies data based on linear decision boundary

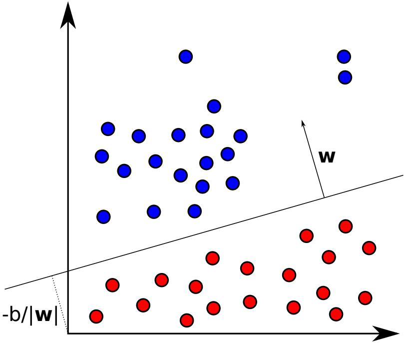
NB: the original perceptron algorithm is a binary classifier (similar to logistic regression but non-probabilistic)

NB: in an N-dimensional feature space, the decision boundary is a ***hyperplane***

⇒ **perceptron**:

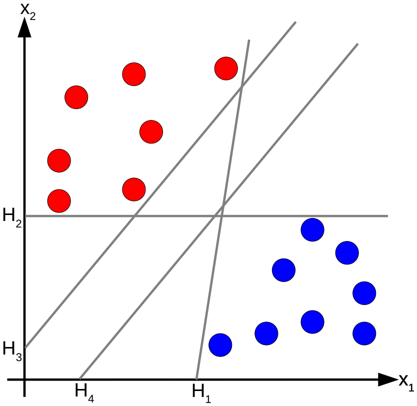
$$\boxed{\hat{y} = \text{sign}(\mathbf{w}^T x + \mathbf{b})}$$

- $\hat{y} \in \{-1, 1\}$: predicted class → *banana or apple*
- $x \in \mathbb{R}^2$: feature vector → *hue, elongation*
- $\mathbf{w} \in \mathbb{R}^2$: **weight vector** → needs to be learned
- $\mathbf{b} \in \mathbb{R}$: **bias** → needs to be learned
- *sign*: sign function returning the sign of a real number



Perceptron classifier

- ⇒ **perceptron**: $\boxed{\hat{y} = \text{sign}(\mathbf{w}^T x + \mathbf{b})}$
- ⇒ problem: multiple "good" boundaries can be found
- ⇒ need to find the *optimal hyperplane*
  - = boundary with maximal margins
  - = perceptron of maximal stability to new inputs



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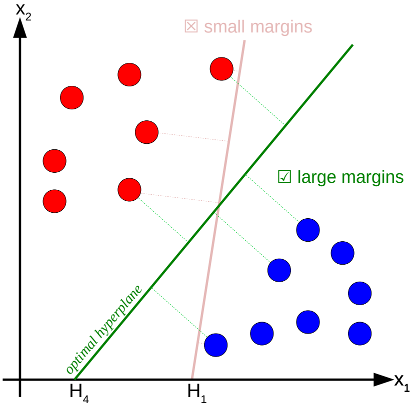

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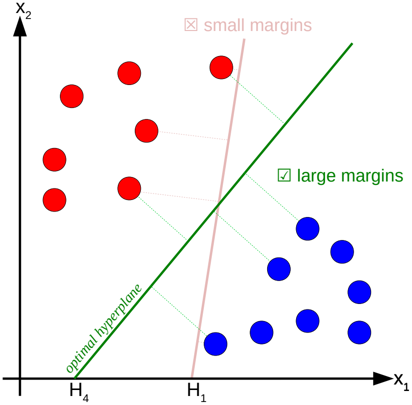
> ⇒ the **Support Vector Machine** (SVM) algorithm will find the *optimal hyperplane* and learn the best *weights* $\mathbf{w}$ and *bias* $\mathbf{b}$ to classify the data

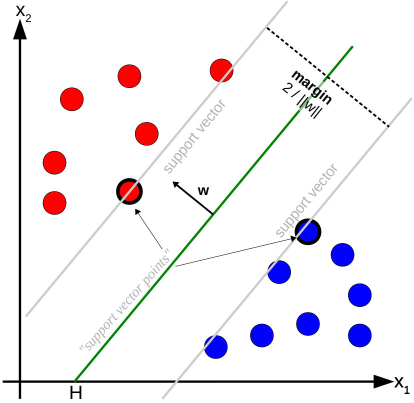


Support Vector Machine (SVM)

⇒ **perceptron**: $\boxed{\hat{y} = \text{sign}(\mathbf{w}^T x + \mathbf{b})}$

⇒ definitions:

- support vector points = points closest to the hyperplane
  (only these points are contributing to the result, other points are not)
- margin = distance between hyperplane & support vector points
  $= \frac{2}{||w||}$



⇒ maximize margin:

$$\max_{w} \frac{2}{||w||}, \text{ subject to } \begin{cases} w^T x_i + b \geq 1 & \text{if } y_i = +1 \\ w^T x_i + b \leq 1 & \text{if } y_i = -1 \end{cases}$$

which is equivalent to:

$$\min_{w} ||w||^2, \text{ subject to } y_i(w^T x_i - b) \geq 1$$

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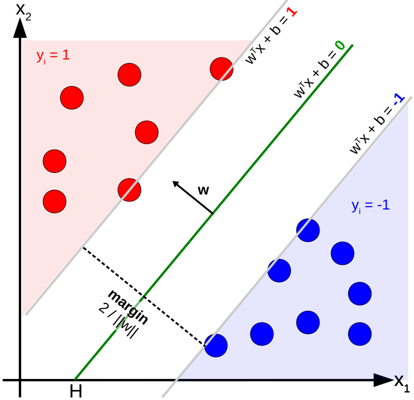


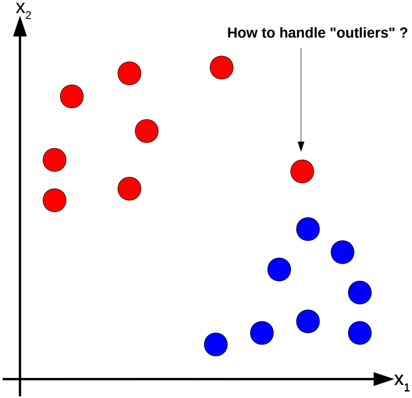
Support Vector Machine (SVM)

⇒ **How can outliers be handled?**

- ⇒ is a hard-margin with 100% accuracy good?
- ⇒ no, allow small errors (**soft-margin**) to favour overall better model
- ⇒ tolerate margin violation & favour large margin boundaries
- ⇒ optimization becomes:

$$\min_{w,\xi_i} ||w||^2 + C\sum_{i}^{N}\xi_i, \text{ subject to } y_i(w^Tx_i - b) \geq 1 - \xi_i$$

$$\text{where: } \begin{cases} C & \text{regularization parameter} \\ & \text{- small C} \Rightarrow \text{constraints easily ignored, large margin} \\ & \text{- large C} \Rightarrow \text{towards hard-margin SVM} \\ \xi_i & \text{slack variable for each data point} \end{cases}$$



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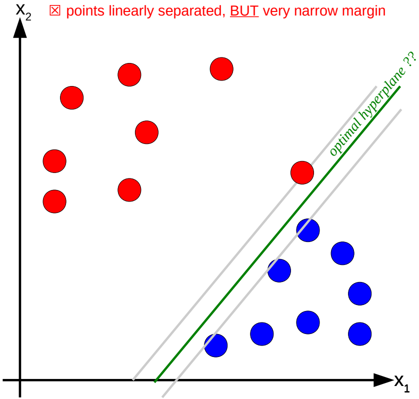
ξ_i slack variable for each data point


☒ points linearly separated, BUT very narrow margin

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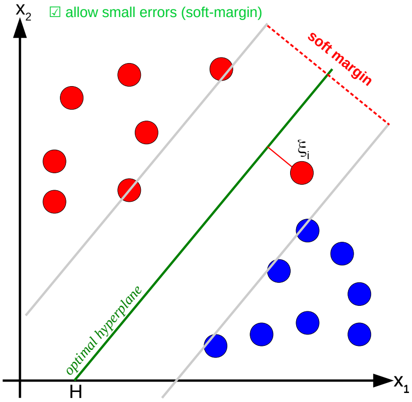

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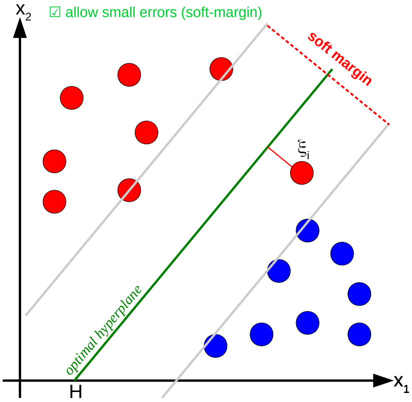

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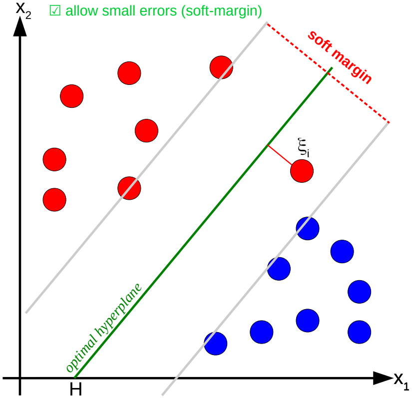


3.2. Support Vector Machine (SVM)

Side note: *reformulating optimization in terms of* ***regularization*** *and* ***loss function*** *(anticipating DL lectures)*

Learning an SVM has been formulated as a constrained optimization problem over w and ξ:

$$\min_{w,\xi_i} ||w||^2 + C \sum_i^N \xi_i \qquad \text{subject to:} \quad y_i(w^T x_i - b) \geq 1 - \xi_i$$

The constraint $y_i(w^T x_i - b) \geq 1 - \xi_i$ can be written more concisely as: $y_i f(x_i) \geq 1 - \xi_i$

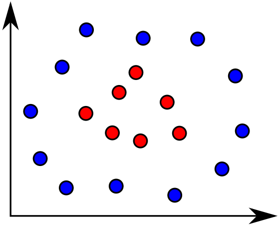
Together with $\xi_i > 0$, it is equivalent to: $\xi_i = max(0, 1 - y_i f(x_i))$

Hence the learning problem is equivalent to the unconstrained optimization problem over w:

$$\min_w \underbrace{||w||^2}_{regularization} + C \sum_i^N \underbrace{max(0,\ 1 - y_i f(x_i))}_{loss\ function\ (Hinge\ loss)}$$

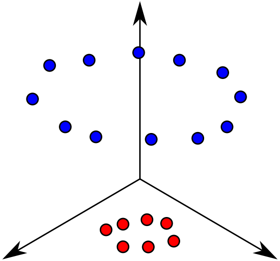
Support Vector Machine (SVM)

- **What if the features x_i are not linearly separable**?



Support Vector Machine (SVM)

- **What if the features x_i are not linearly separable**?
  $\Rightarrow$ compute new features $x_i \mapsto \phi(x)$
  $\phi(x)$ is a **feature map**, mapping x to $\phi(x)$ where data is separable



Support Vector Machine (SVM)

- **What if the features x_i are not linearly separable**?
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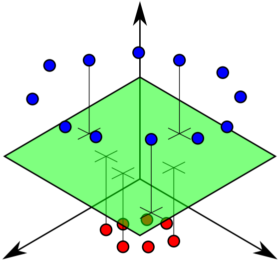
  $\Rightarrow$ solve for $\mathbf{w}$ in high dimensional feature space



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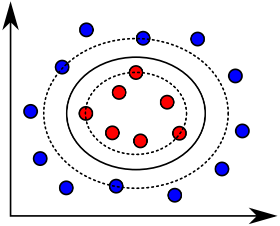
  $\Rightarrow$ data not lineary-seperable in original feature space become separable



3.2. Support Vector Machine (SVM)

Kernel trick

The Representer Theorem states that the solution **w** can be written as a linear combination of the training data:

$$w = \sum_{j=1}^{N} \alpha_j y_j x$$

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The linear classifier can therefore be reformulated as:

$$\begin{aligned} f(x) &= w^T x + b \\ &= \sum_{i}^{N} \alpha_i y_i (x_i^T x) + b \end{aligned}$$

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Using the feature map $\phi(x)$, it can be reformulated as:

$$\begin{aligned} f(x) &= \sum_{i}^{N} \alpha_i y_i (\phi(x_i)^T \phi(x)) + b \\ &= \sum_{i}^{N} \alpha_i y_i k(x_i, x) + b \end{aligned}$$

where $k(x_i, x)$ is known as a **Kernel**

Kernel trick

- Classifier can be learnt and applied without explicitly computing $\phi(x)$
- All that is required is the kernel $k(x, x')$
- Multiple kernels exist:
  - linear kernels: $k(x, x') = x^T x'$
    → *very fast and easy to train, but very simple*
  - polynomial kernels: $k(x, x') = (1 + x^T x')^d$
    → *contains all polynomial terms up to degree d*
  - gaussian kernels: $k(x, x') = exp(-||x - x'||^2 / 2\sigma^2)$ *(RBF kernel)*
    → *kernel very powerful and most often used*

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1. Introduction
2. Principal Component Analysis (PCA)
3. Classification algorithms (perceptron + SVM)
4. Exercise

Exercise

EXERCISE:

classify land-use in satellite images (Sentinel-2) using PCA and SVM

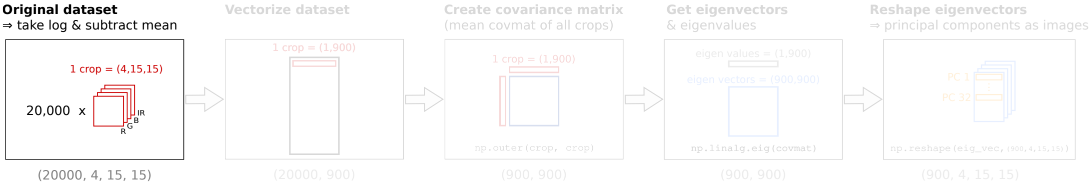
PCA dimensionality reduction

train SVM & apply

land-use classification

Exercise

Part 1: apply PCA on satellite image crops



Original dataset
⇒ take log & subtract mean

(20000, 4, 15, 15)

Vectorize dataset

(20000, 900)

Create covariance matrix
(mean covmat of all crops)

`np.outer(crop, crop)`

(900, 900)

Get eigenvectors
& eigenvalues

`np.linalg.eig(covmat)`

(900, 900)

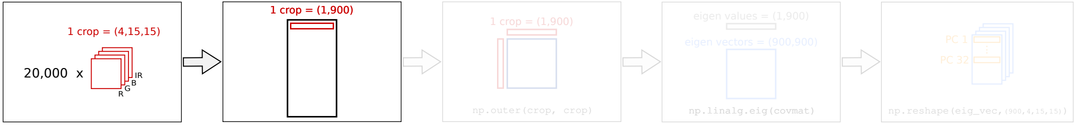
Reshape eigenvectors
⇒ principal components as images

`np.reshape(eig_vec,(900,4,15,15))`

(900, 4, 15, 15)

Exercise

Part 1: apply PCA on satellite image crops



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Get eigenvectors
& eigenvalues
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Reshape eigenvectors
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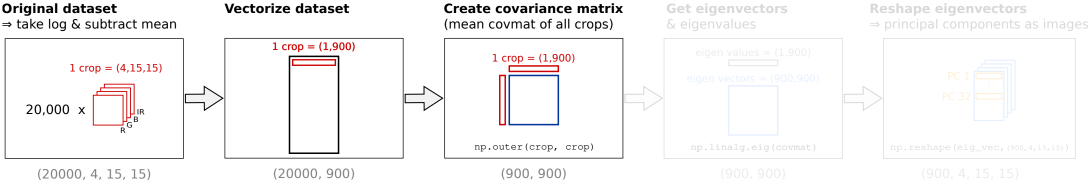
(900, 4, 15, 15)

Exercise

Part 1: apply PCA on satellite image crops



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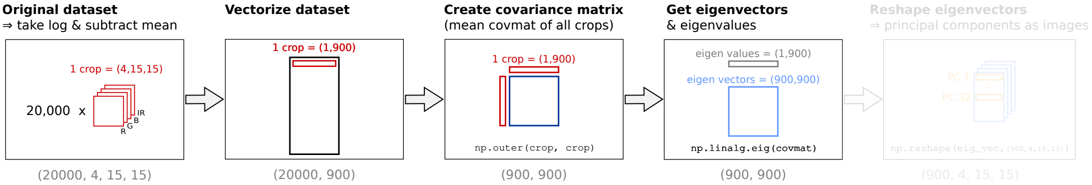
Exercise

Part 1: apply PCA on satellite image crops



Original dataset
⇒ take log & subtract mean

1 crop = (4,15,15)

20,000 x

IR

B

G

R

`(20000, 4, 15, 15)`

Vectorize dataset

1 crop = (1,900)

`(20000, 900)`

Create covariance matrix
(mean covmat of all crops)

1 crop = (1,900)

`np.outer(crop, crop)`

`(900, 900)`

Get eigenvectors
& eigenvalues

eigen values = (1,900)

eigen vectors = (900,900)

`np.linalg.eig(covmat)`

`(900, 900)`

Reshape eigenvectors
⇒ principal components as images

PC 1

⋮

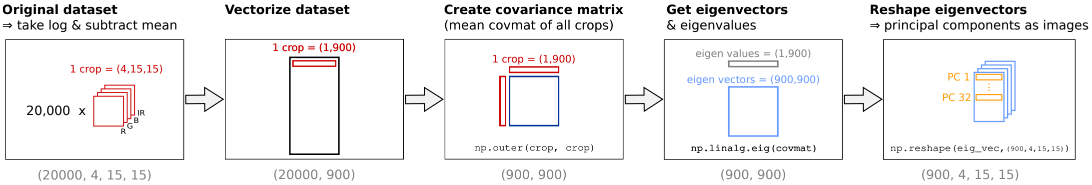
PC 32

`np.reshape(eig_vec,(900,4,15,15))`

`(900, 4, 15, 15)`

Exercise

Part 1: apply PCA on satellite image crops



Original dataset
⇒ take log & subtract mean

20,000 x 1 crop = (4,15,15)

(20000, 4, 15, 15)

Vectorize dataset

1 crop = (1,900)

(20000, 900)

Create covariance matrix
(mean covmat of all crops)

1 crop = (1,900)

`np.outer(crop, crop)`

(900, 900)

Get eigenvectors
& eigenvalues

eigen values = (1,900)

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Reshape eigenvectors
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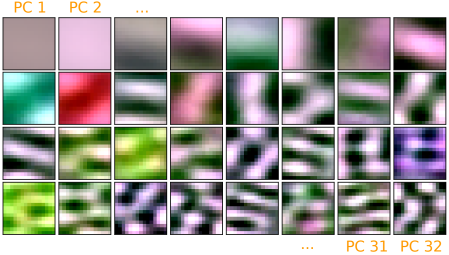
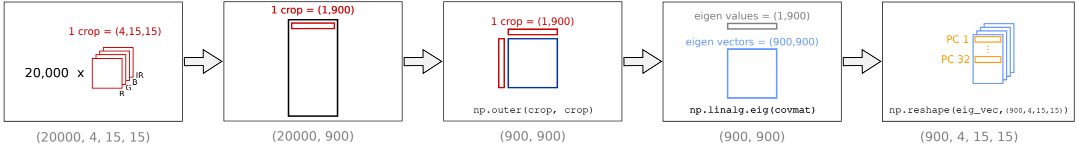


Exercise

Part 1: apply PCA on satellite image crops



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Reshape eigenvectors
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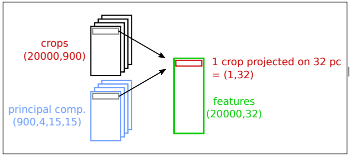
`np.reshape(eig_vec, (900,4,15,15))`

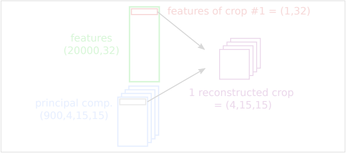
(900, 4, 15, 15)

Compute features
⇒ project each crop on first 32 pc



```
for i in range(20000):   #loop crops
   for j in range(32):   #loop pcs
      features[i,j] = np.dot(crop[i,:::], pc[j,:::])
                    = (900,1) @ (1,900)
                    = (scalar)
```

Reconstruct crops
⇒ reconstruct crop from its 32 features & 32 first pcs

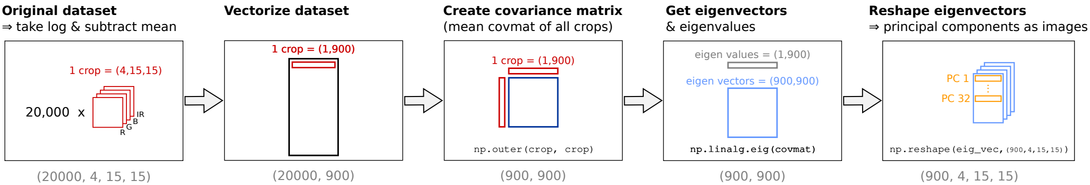


Reconstruction crop #1:

```
reconstruction = mean_crops   #(4, 15, 15)
for i in range(32):           #loop crop features/pcs
   reconstruction += features[0,i] * pc[i,:::]
                   = (scalar) * (4,15,15)
                   = (4,5,15)
```

Exercise

Part 1: apply PCA on satellite image crops



Original dataset
⇒ take log & subtract mean

1 crop = (4,15,15)

20,000 x

IR B G R

(20000, 4, 15, 15)

Vectorize dataset

1 crop = (1,900)

(20000, 900)

Create covariance matrix
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1 crop = (1,900)

`np.outer(crop, crop)`

(900, 900)

Get eigenvectors
& eigenvalues

eigen values = (1,900)

eigen vectors = (900,900)

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Reshape eigenvectors
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PC 1
⋮
PC 32

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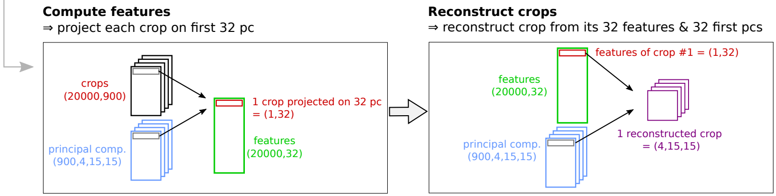

Compute features
⇒ project each crop on first 32 pc

crops
(20000,900)

principal comp.
(900,4,15,15)

1 crop projected on 32 pc
= (1,32)

features
(20000,32)

```
for i in range(20000):    #loop crops
   for j in range(32):    #loop pcs
      features[i,j] = np.dot(crop[i,:::], pc[j,:::])
                    = (900,1) @ (1,900)
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```

Reconstruct crops
⇒ reconstruct crop from its 32 features & 32 first pcs

features of crop #1 = (1,32)

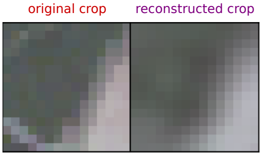
features
(20000,32)

principal comp.
(900,4,15,15)

1 reconstructed crop
= (4,15,15)

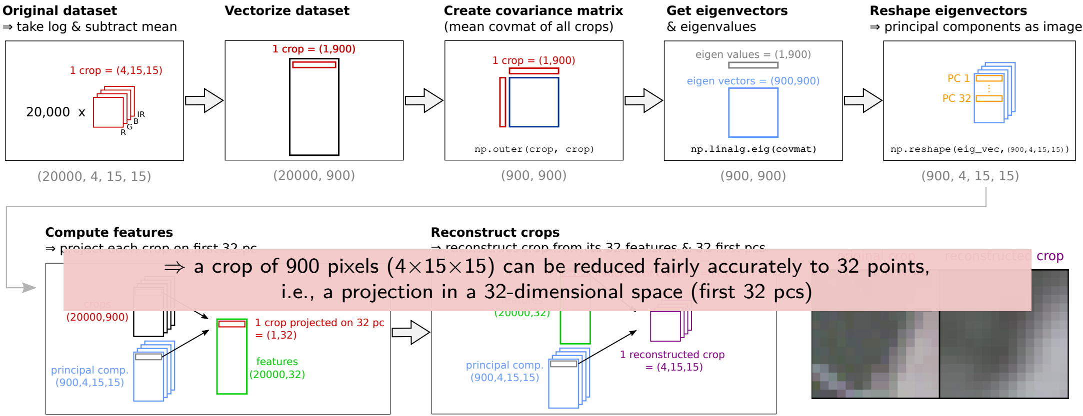
Reconstruction crop #1:

```
reconstruction = mean_crops   #(4, 15, 15)
for i in range(32):           #loop crop features/pcs
   reconstruction += features[0,i] * pc[i,:::]
                   = (scalar) * (4,15,15)
                   = (4,5,15)
```



Exercise

Part 1: apply PCA on satellite image crops



⇒ a crop of 900 pixels (4×15×15) can be reduced fairly accurately to 32 points,
i.e., a projection in a 32-dimensional space (first 32 pcs)

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for i in range(20000):   #loop crops
   for j in range(32):   #loop pcs
      features[i,j] = np.dot(crop[i,:::], pc[j,:::])
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Reconstruction crop #1:

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reconstruction = mean_crops    #(4, 15, 15)
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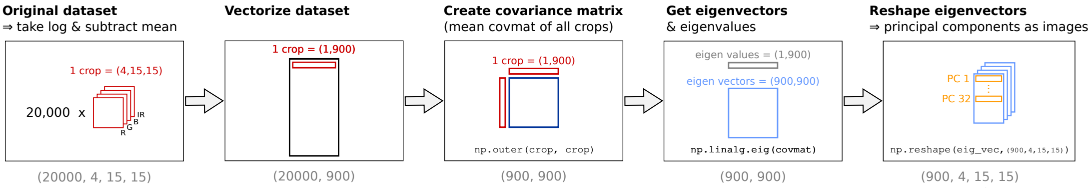
Exercise

Part 1: apply PCA on satellite image crops



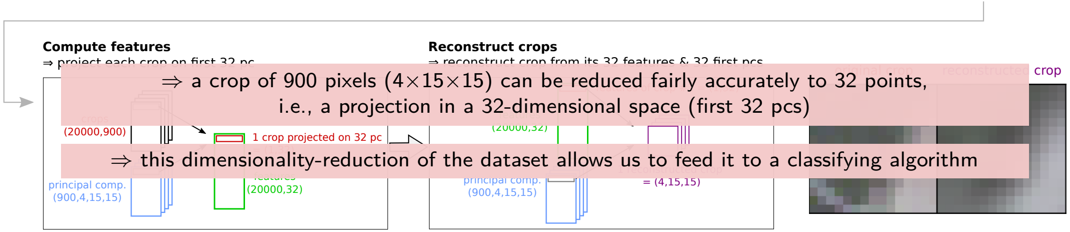
Original dataset
⇒ take log & subtract mean

Vectorize dataset

Create covariance matrix
(mean covmat of all crops)

Get eigenvectors
& eigenvalues

Reshape eigenvectors
⇒ principal components as images



Compute features
⇒ project each crop on first 32 pc

Reconstruct crops
⇒ reconstruct crop from its 32 features & 32 first pcs

⇒ a crop of 900 pixels (4×15×15) can be reduced fairly accurately to 32 points,
i.e., a projection in a 32-dimensional space (first 32 pcs)

⇒ this dimensionality-reduction of the dataset allows us to feed it to a classifying algorithm

```
for i in range(20000):   #loop crops
   for j in range(32):   #loop pcs
      features[i,j] = np.dot(crop[i,:::], pc[j,:::])
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Reconstruction crop #1:

```
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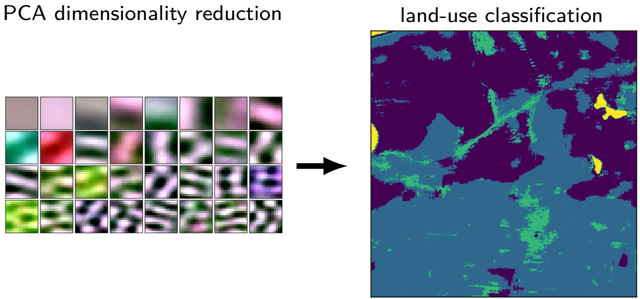
                   = (scalar) * (4,15,15)
                   = (4,5,15)
```

Exercise

Part 2: train SVM on principal components and apply to classify full image



PCA dimensionality reduction

land-use classification