Lecture 05
Homography

2024-09-11

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Introduction:

Image transformations:

$$g(x, y) = T[f(x, y)]$$

where:

- f(x, y) is the input image
- g(x, y) is the output image
- T is an operator

Previous lecture(s):

- **point operators**
  ⇒ transform pixel value f(x,y), ignoring surrounding pixels → *neighborhood of T=1x1 pixel*
  ⇒ intensity transformation functions (EX: change image contrast with $g(x, y) = f(x, y)^2$)
- **local operators**
  ⇒ transform pixel value f(x,y) based on surrounding pixels → *neighborhood of T>1x1 pixel*
  ⇒ linear operators (filtering with convolutions), morphological operators (filtering with morphology)

Today's lecture:

- **geometrical operators**
  ⇒ geometrical operators **do not** change pixel value, instead "**move**" it to a new position

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Homography is used to transform an image from one projective plane to another

Applications in image processing:

- image stitching (e.g., mosaics and panoramas)
- image registration (e.g., "fuse" datasets in unique coordinate frame)
- image warping (e.g., change image perspective, correct lense distortion, etc.)
- Structure from Motion (SfM) (i.e., 3D reconstruction from multiple images)
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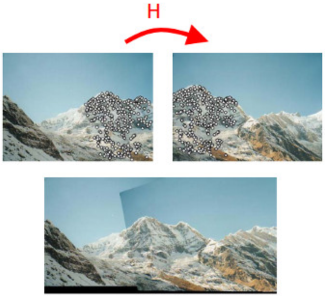

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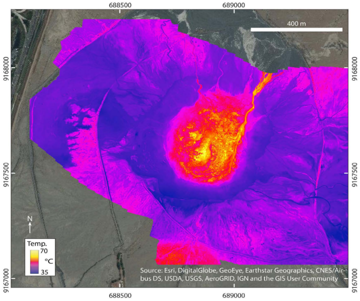

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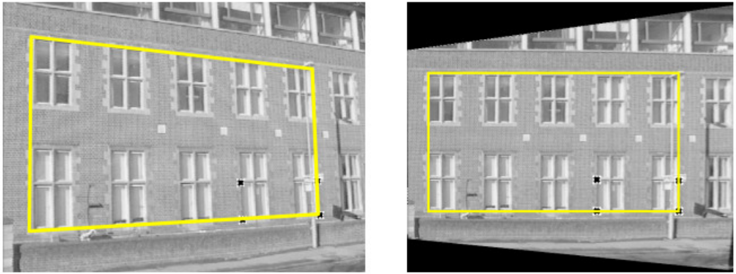

from Hartley & Zisserman

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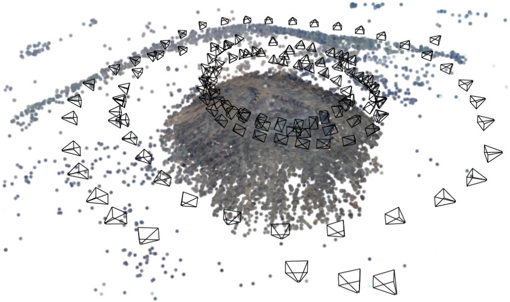

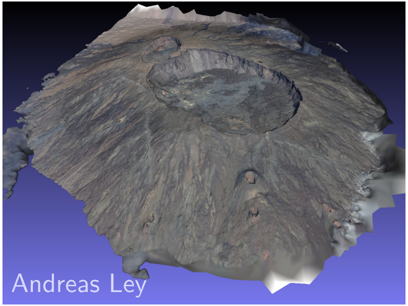

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Geometric transformations map points from one space to another:

$$(x', y') = f(x, y)$$

⇒ in linear algebra, linear transformations can be represented by matrix operations:

$$X' = MX \tag{1}$$

where:

- $X = \begin{bmatrix} x \\ y \end{bmatrix}$ = original pixel coordinates
- $X' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ = transformed pixel coordinates
- $\mathrm{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ = **transformation matrix**

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The matrix equation:

$$X' = MX$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Can we written as a linear system of equations:

$$\begin{cases} x' = ax + by \\ y' = cx + dy \end{cases}$$

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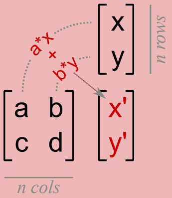
Reminder: matrix multiplication



2.2. definition

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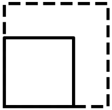
The transformation matrix M will determine the type of geometric transformation.

Example 1: scale points?

scaling



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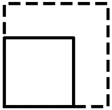

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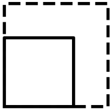

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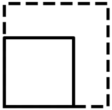

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$\Rightarrow$ the point with coordinates $\begin{bmatrix} x \\ y \end{bmatrix}$ is transformed to coordinates $\begin{bmatrix} x' \\ y' \end{bmatrix}$ using the matrix multiplication:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$
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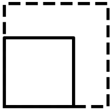

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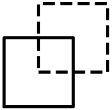
⇒ in Python this translates as:

```
import numpy as np
X = np.array([1, 1]).T            # original coordinates (x, y)
sx, sy = 2, 2                     # scaling factors
M = np.array([[sx,0], [sy,2]])   # transformation matrix
X_prime = M @ X                   # transformed coordinates (x', y') from matrix multiplication
# returns: X_prime = array([2, 2])
```

Example 2: translate points?

translation



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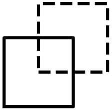

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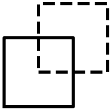

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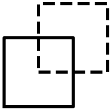

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$\Rightarrow$ add a component to the coordinates: redefine $X = \begin{bmatrix} x \\ y \end{bmatrix}$ as $\overline{X} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ = "augmented vector"

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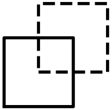

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$\Rightarrow$ the transformation matrix to translate can now be defined as: $M = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

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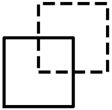

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⇒ hence the transformation coordinates can be calculated from:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1x + 0y + 1t_x \\ 0x + 1y + 1t_y \\ 0x + 0y + 1 \end{bmatrix}$$

$$= \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$$

2.2. definition

Homogeneous & Heterogeneous coordinates

- **heterogeneous coordinates** (a.k.a. **Cartesian**, **Euclidean**)
  ⇒ coordinates used to represent points in the regular Euclidean space: $[x, y]$ in 2D space, $[x, y, z]$ in 3D space
- **homogeneous coordinates**
  ⇒ extension of the heterogeneous coordinates using *augmented vectors*
  ⇒ used to represent points in a higher-dimensional space, making transformations (e.g. translation, rotation, scaling, projection) possible in a consistent mathematical framework

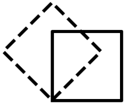
	heterogeneous		**homogeneous**
point in 2D space:	$\begin{bmatrix} x \\ y \end{bmatrix}$	$\rightarrow$	$\begin{bmatrix} x \\ y \\ w \end{bmatrix}$
point in 3D space:	$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$	$\rightarrow$	$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$

where w is the "homogeneous coordinate":

- if $w = 1$: $[x, y, 1]$ represents a point in Cartesian coordinates (x,y)
- if $w \neq 1$: $[x, y, w]$ represents a point in a scaled version of Cartesian coordinates
  $\rightarrow$ actual Cartesian coordinates are obtained by dividing by w: $[x/w, y/w]$

Example 3: other simple transformations?

rotation



$$\begin{cases} x' = x * cos\theta - y * sin\theta \\ y' = x * cos\theta + y * sin\theta \end{cases}$$

$$M = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(counter-clockwise rotation from x-axis)

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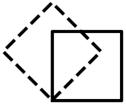

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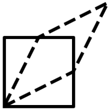
shear
(= **skew**)



$$\begin{cases} x' = x + s_v * y \\ y' = x * s_h + y \end{cases}$$

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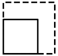
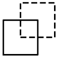
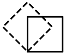
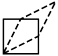
"Primary" 2D transformations:

Transformation Type	Transformation Matrix M	Pixel Mapping Equation	
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = y$	
Scaling	$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = s_x * x$ $y' = s_y * y$	
Translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + t_x$ $y' = y + t_y$	
Rotation (counter-clockwise about origin)	$\begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x * cos\theta - y * sin\theta$ $y' = x * cos\theta + y * sin\theta$	
Shear (a.k.a. Skew)	$\begin{bmatrix} 1 & s_h & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + s_v * y$ $y' = x * s_h + y$	

"Composite" 2D transformations ⇒ concatenation of "primary" transformations

Example: **Euclidean transformation** (a.k.a. "rigid transform", or "motion")

⇒ rotation *(transformation 1)* followed by a translation *(transformation 2)*

⇒ the transformation matrix is therefore defined as: $M = M_{translation} \cdot M_{rotation} = transform\ 2 \cdot transform\ 1$
important: transformation concatenation order is from right to left, think like $f(g(x))$

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M &= transform\ 2 \cdot transform\ 1 \\
&= M_{translation} \cdot M_{rotation} \\
&= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} cos\theta & -sin\theta & t_x \\ sin\theta & cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}
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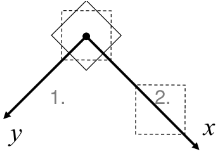

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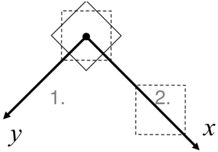
&= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} cos\theta & -sin\theta & t_x \\ sin\theta & cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$



order matters !

$$
\begin{aligned}
M &\neq transformation\ 1 \cdot transformation\ 2 \\
&\neq M_{rotation} \cdot M_{translation} \\
&\neq \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \\
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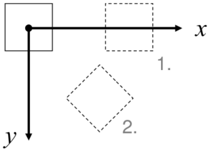
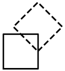
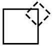
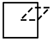
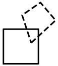


2.2. definition

⇒ in Python this translates as:

```python
import numpy as np

# set rotation transformation matrix
angle = np.deg2rad(45)
R = np.array([
    [np.cos(angle), -np.sin(angle), 0],
    [np.sin(angle),  np.cos(angle), 0],
    [0, 0, 1]])

# set translation transformation matrix
tx, ty = 1, .5
T = np.array([
    [1, 0, tx],
    [0, 1, ty],
    [0, 0, 1]])

# set original coordinates
X = np.array([
    [0, 0, 1],  # point 1 (x,y,w)
    [1, 0, 1],  # point 2 (x,y,w)
    [1, 1, 1],  # point 3 (x,y,w)
    [0, 1, 1]]) # point 4 (x,y,w)

# get euclidean transformation matrix as (1) rotation followed by (2) translation
M = T @ R

# get transformed coordinates (x', y')
X_prime = M @ X.T
```

"Composite" 2D transformations:

Transformation Type	Transformation Matrix M	Pixel Mapping Equation	
Euclidean transformation (a.k.a. "rigid transform", or "motion") = rotation → translation	$\begin{bmatrix} cos\theta & -sin\theta & tx \\ sin\theta & cos\theta & ty \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x * cos\theta - y * sin\theta + tx$ $y' = x * sin\theta + y * cos\theta + ty$	
Similarity transformation = rotation → translation → scale	$\begin{bmatrix} a & -b & tx \\ b & a & ty \\ 0 & 0 & 1 \end{bmatrix}$	$x' = s * x * cos\theta - s * y * sin\theta + tx$ $y' = s * x * sin\theta + s * y * cos\theta + ty$	
Affine transformation = similarity → shear	$\begin{bmatrix} a & b & tx \\ c & d & ty \\ 0 & 0 & 1 \end{bmatrix}$	$x' = sx * x * cos(\theta) - sy * y * sin(\theta + shear) + tx$ $y' = sx * x * sin(\theta) + sy * y * cos(\theta + shear) + ty$	
Projective transformation (a.k.a. **homography**)	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$	encompasses rotation, scaling, shear and perspective	

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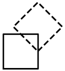
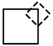
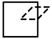
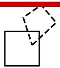
Euclidean transformation (a.k.a. “rigid transform”, or “motion”) = rotation $\rightarrow$ translation	$\begin{bmatrix} cos\theta & -sin\theta & tx \\ sin\theta & cos\theta & ty \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x * cos\theta - y * sin\theta + tx$ $y' = x * sin\theta + y * cos\theta + ty$	
Similarity transformation = rotation $\rightarrow$ translation $\rightarrow$ scale	$\begin{bmatrix} a & -b & tx \\ b & a & ty \\ 0 & 0 & 1 \end{bmatrix}$	$x' = s * x * cos\theta - s * y * sin\theta + tx$ $y' = s * x * sin\theta + s * y * cos\theta + ty$	
Affine transformation = similarity $\rightarrow$ shear	$\begin{bmatrix} a & b & tx \\ c & d & ty \\ 0 & 0 & 1 \end{bmatrix}$	$x' = sx * x * cos(\theta) - sy * y * sin(\theta + shear) + tx$ $y' = sx * x * sin(\theta) + sy * y * cos(\theta + shear) + ty$	
Projective transformation (a.k.a. **homography**)	$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$	encompasses rotation, scaling, shear and perspective	

2.2. definition

⇒ the homography matrix H has 8 degrees of freedom (DOF):

$$H = \begin{bmatrix} H_{00} & H_{01} & H_{02} \\ H_{10} & H_{11} & H_{12} \\ H_{20} & H_{21} & 1 \end{bmatrix}$$

⇒ estimating these parameters is key to transforming from one coordinate system to another

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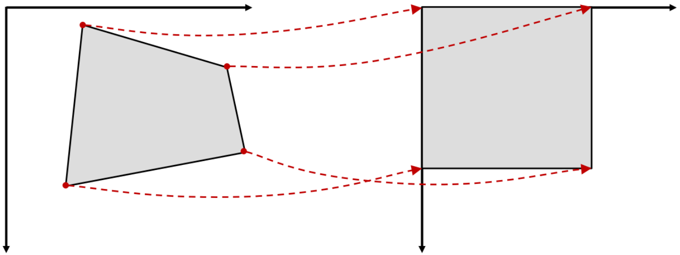

EX1: digital planar rectification

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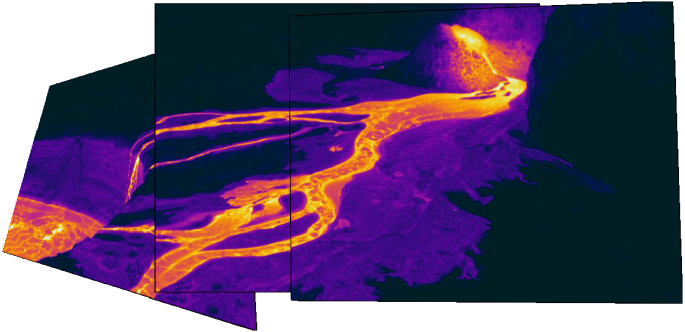

EX2: panorama creation

How do we estimate these 8 parameters?

⇒ the **Direct Linear Transformation** (DLT) is an algorithm for computing H

- Given: at least $n \geqslant 4$ point pairs $X_i \rightarrow X_i'$ (where X_i = coordinates in image 1, X_i' = coordinates in image 2)
- Wanted: 3×3 homography matrix H (8 DOF), for which $X_i' = HX_i$ holds

1. Reformulate the general projective transformation into a linear homogeneous equation system

⇒ reformulate $X' = HX$ into $Ah = 0$
⇒ will allow us to solve for the unknowns h using SVD (Singular Value Decomposition)

General projective transformation:

$$X' = HX$$

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} H_{00} & H_{01} & H_{02} \\ H_{10} & H_{11} & H_{12} \\ H_{20} & H_{21} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Write as linear equation system:

$$\begin{cases} x' = H_{00}x + H_{01}y + H_{02}w \\ y' = H_{10}x + H_{11}y + H_{12}w \\ w' = H_{20}x + H_{21}y + H_{22}w \end{cases}$$

Convert back from homogeneous to Euclidean coordinates by dividing with w', and move all terms to the left:

$$\frac{x'}{w'} - \frac{H_{00}x + H_{01}y + H_{02}}{H_{20}x + H_{21}y + H_{22}} = 0$$

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2.3. estimating the homography matrix

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1. (continued)

Multiplying by the denominator $(H_{20}x + H_{21}y + H_{22})$ yields:

$$\frac{x'}{w'}(H_{20}x + H_{21}y + H_{22}) - H_{00}x - H_{01}y - H_{02} = 0$$
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Which can be written as the system:

$$\begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & \frac{x'x}{w'} & \frac{x'y}{w'} & \frac{x'}{w'} \\ 0 & 0 & 0 & -x & -y & -1 & \frac{y'x}{w'} & \frac{y'y}{w'} & \frac{y'}{w'} \end{bmatrix} \begin{bmatrix} H_{00} \\ H_{01} \\ \vdots \\ H_{21} \\ H_{22} \end{bmatrix} = 0$$

We now have to solve the homogeneous set of linear equations:

$$Ah = 0$$

where:

- A is the "*design matrix*", in which each point pair n fills 2 rows (2 observations per point: x and y coordinates) $\Rightarrow$ shape $= 2n \times 9$
  NB: (x_1, y_1) and (x'_1, y'_1) refer to coordinates of the point pair #1, in image 1 and 2 respectively

$$A = \begin{bmatrix} -x_1 & -y_1 & -1 & 0 & 0 & 0 & \frac{x'_1 x_1}{w'} & \frac{x'_1 y_1}{w'} & \frac{x'_1}{w'} \\ 0 & 0 & 0 & -x_1 & -y_1 & -1 & \frac{y'_1 x_1}{w'} & \frac{y'_1 y_1}{w'} & \frac{y'_1}{w'} \\ & \vdots & & \vdots & & \vdots & & \vdots & \end{bmatrix}$$

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- h is the vector of unknowns: $h = \begin{bmatrix} H_{00} & H_{01} & H_{02} & H_{10} & H_{11} & H_{12} & H_{20} & H_{21} & H_{22} \end{bmatrix}^T$

2. Solve the homogeneous equation system with Singular Value Decomposition (SVD)

Note: SVD is generally used for finding solutions of over-determined systems.

The “singular value decomposition” of matrix A is a factorization of the form:

$$A = UDV^T$$

where:
- the diagonal elements of D (arranged to be non-negative and in decreasing order of magnitude), are called **singular values**
- the matrices U and V are called left and right **singular vectors** respectively

⇒ **the least squares solution is found as the last row of the matrix V of the SVD**

⇒ this translate in Python as:

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import numpy as np
U,S,V = np.linalg.svd(A)     # singular value decomposition of A
h = V[8]                     # least squares solution found as the last row of V
H = h.reshape((3,3))         # reshape into 3x3 homography matrix
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3. Conditioning & Unconditioning of points

In order to stabilize the solution, once the points are selected, they need to be conditioned (i.e. before creating the design matrix A and solving for H)

⇒ the points are conditioned so that they have *zero mean* and *unit standard deviation*:

- *zero mean* ⇒ the centroid of the points is at the origin (0,0)
- *unit standard deviation* ⇒ standard deviation (spread) of points is equal to 1 (achieved by subtracting the mean and dividing by the std. dev.)

point selection

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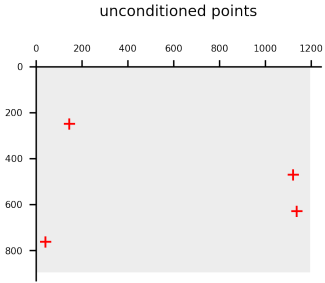

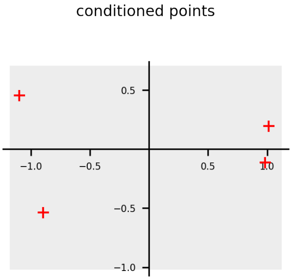

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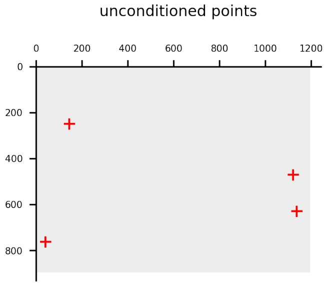

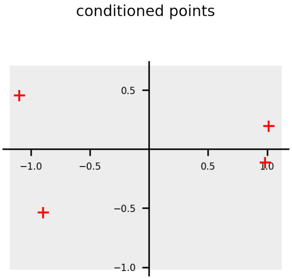

⇒ can be done with the "*conditioning matrix*" C (consisting of scaling & translation to origin):

$$C = \begin{bmatrix} s & 0 & tx \\ 0 & s & ty \\ 0 & 0 & 1 \end{bmatrix}$$

where: $s = \frac{1}{max([std_x, std_y])}$, $t_x = \frac{-mean_x}{max([std_x, std_y])}$, and $t_y = \frac{-mean_y}{max([std_x, std_y])}$

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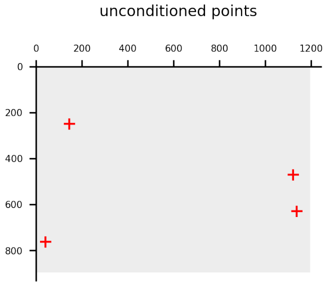

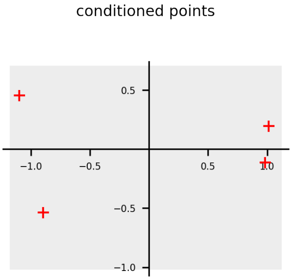

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⇒ a condition matrix is constructed for each image, and conditioned coordinates are then calculated as: $\widetilde{X} = C_1 X$ and $\widetilde{X'} = C_2 X'$

3. (continued)

The solved H matrix is in conditioned coordinates, so it must be "deconditioned" before it can be used:

⇒ conditioned homography matrix: $\widetilde{H} = \begin{bmatrix} \widetilde{H}_{00} & \widetilde{H}_{01} & \widetilde{H}_{02} \\ \widetilde{H}_{10} & \widetilde{H}_{11} & \widetilde{H}_{12} \\ \widetilde{H}_{20} & \widetilde{H}_{21} & \widetilde{H}_{22} \end{bmatrix}$

⇒ unconditioned homography matrix can be calculated as: $H = C_2^{-1}\widetilde{H}C_1 = \begin{bmatrix} H_{00} & H_{01} & H_{02} \\ H_{10} & H_{11} & H_{12} \\ H_{20} & H_{21} & \widetilde{H}_{22} \end{bmatrix}$

Lastly, H is normalized by the last element H_{22} ("homogeneous coordinate"), and is ready to be used!

2.4. image warping

Then what?

$\Rightarrow$ applying the transformation matrix H on an image is called **warping**

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Case examples:

1. Projection rectification

⇒ use the estimated homography to change the projection of an image

Original

Transformed

Then what?

⇒ applying the transformation matrix H on an image is called **warping**

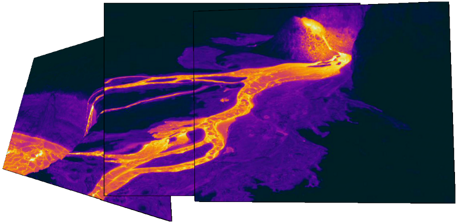
Case examples:

1. Projection rectification

⇒ use the estimated homography to change the projection of an image

2. Panorama stitching

⇒ use the estimated homography(ies) to adapt image(s) to a central image



3.1. interest points

We have seen that homographies can be computed directly from corresponding points in two images:

$\Rightarrow$ since a full projective transformation (homography) has 8 degrees of freedom, and since each point correspondence gives two equations, (one each for the x and y coordinates), $\geqslant 4$ points correspondences are needed to compute H

However manually selecting corresponding points is cumbersome and not scalable!

Solution? Identify **interest points** in image(s)

$\Rightarrow$ provide distinctive image points

$\Rightarrow$ used in tracking (optical flow), object recognition, Structure from Motion

Example of most common interest points:

- Corner Detectors (e.g., Harris, Shi-Tomasi, Förstner, etc.)
- Blob and Ridge Detectors (e.g., LoG, DoG, Hessian, etc.)
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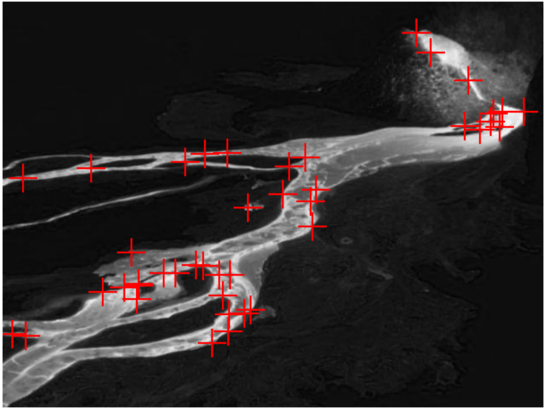
we will discuss in more detail about interest points and features during the next lecture

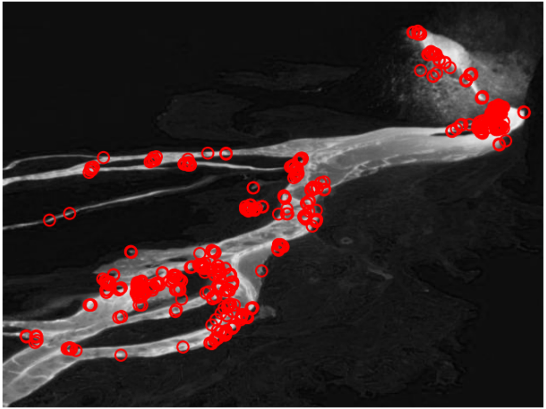
3.2. generate panorama with interest points + RANSAC

Example: **Harris corners** & **ORB features** detected automatically in an image

Harris corners



ORB features



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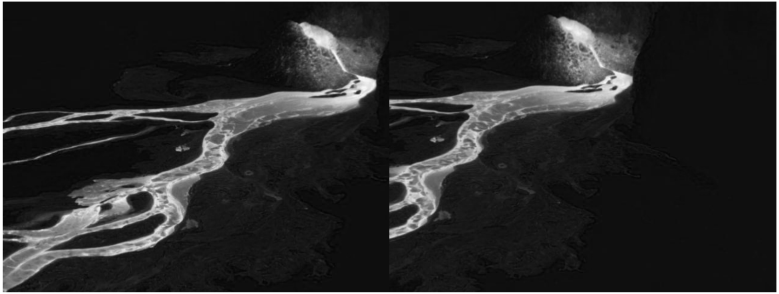

1. take images with overlap

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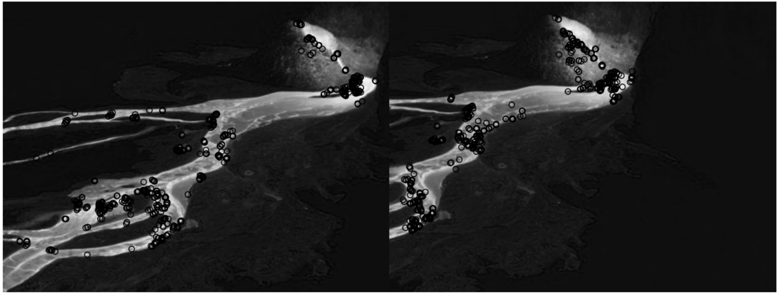

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2. detect ORB features in both images seperately

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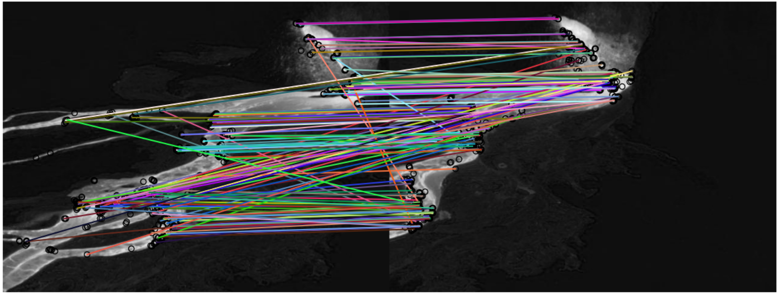

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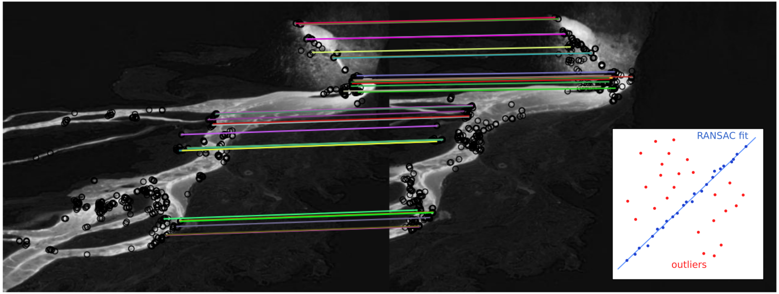

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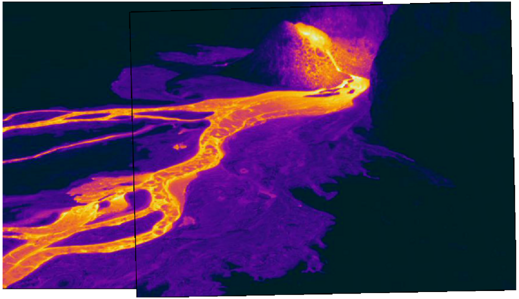

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4. remove outliers with RANSAC (robust iterative regression algorithm, resistant to outliers)
5. estimate homography and warp

Exercises !