Lecture 03
Image Filtering

2024-08-28

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Introduction

The image transformations discussed so far are based on the expression:

$$g(x, y) = T[f(x, y)]$$

where:

- f(x, y) is an input image
- g(x, y) is the output image
- T is an operator on f defined over a neighborhood of point (x, y)

Previous lecture:

$\Rightarrow$ the operator T was applied to individual pixels ("Point Operations"), i.e. neighborhood = 1x1 pix

$\Rightarrow$ the function is an intensity transformation function, to change image contrast, etc.

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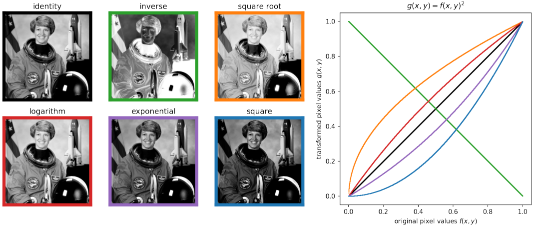

Introduction

Today: filtering!

⇒ Purpose: blur, sharpen, remove noise, filter frequencies, etc.

⇒ Approaches:

1. **spatial domain filtering**
   - the neighborhood is >1 pixel ("Point Processing" → "Neighborhood Processing")
   - spatial filtering modifies an image by replacing the value of each pixel by a function of the values of the pixel and its neighbors
   - if the operation performed on the image pixels is linear, then the filter is called a linear spatial filter
   - spatial filters are applied by **convolution**
2. **frequency domain filtering**
   - the **2D direct Fourier transform** is applied to extract image frequencies
   - the amplitude spectrum can be band-passed to filter certain frequencies
   - the inverse 2D direct Fourier transform is used to reconstruct the filtered image

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Linear spatial filter

⇒ sum-of-products operation between an **input image f(x,y)** and a **filter kernel w**

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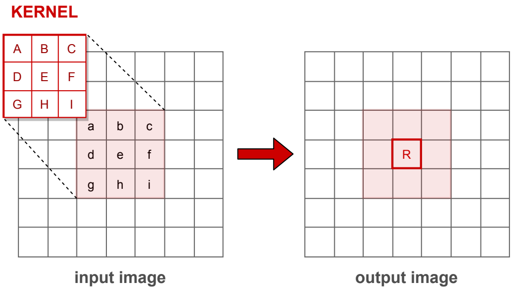

R = A*a + B*b + ... + H*h + I*i

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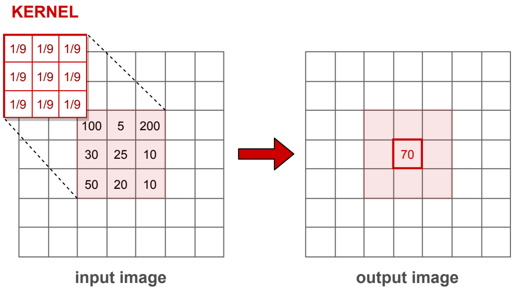

R = 1/9*100 + 1/9*5 + ... + 1/9*20 + 1/9*10

R = 70

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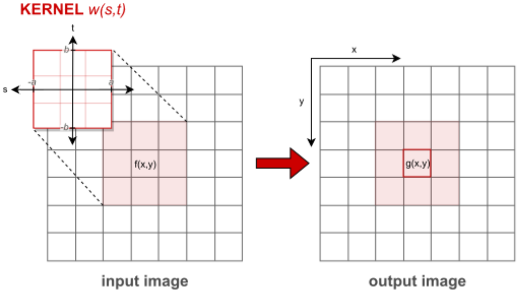

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) \cdot f(x-s, y-t)$$

where a and b define an odd-shape kernel size (m=2a+1, n=2b+1)

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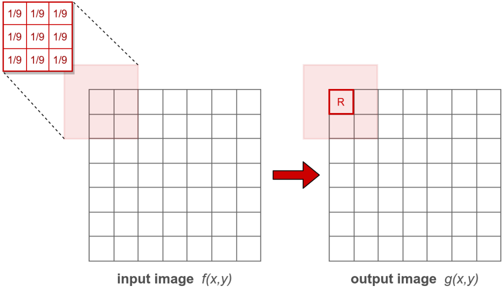

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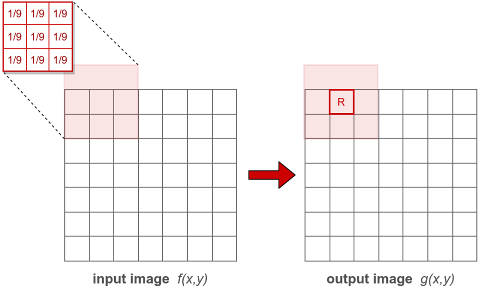

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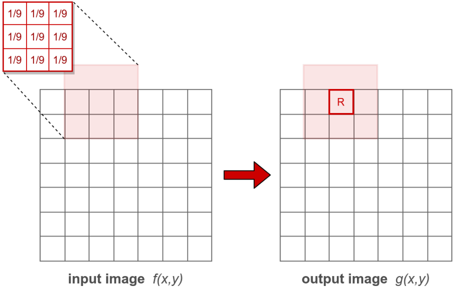

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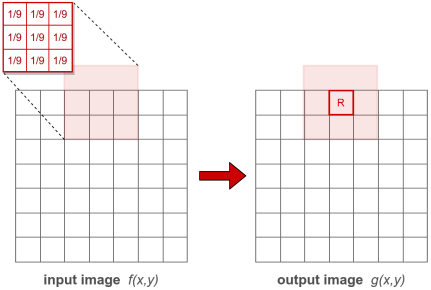

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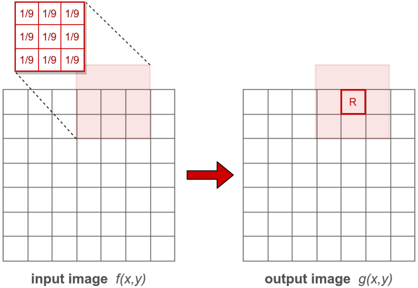

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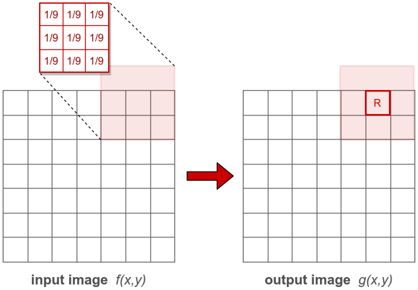

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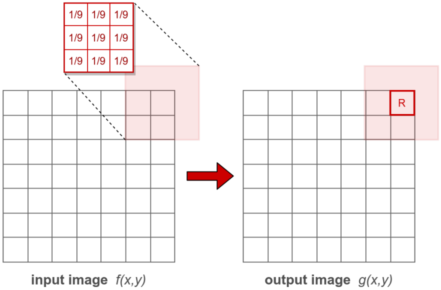

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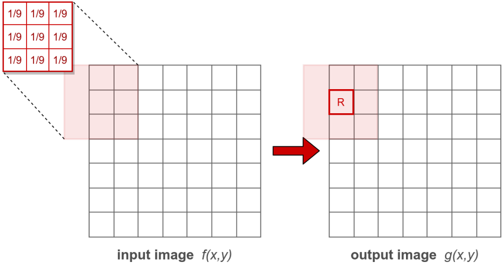

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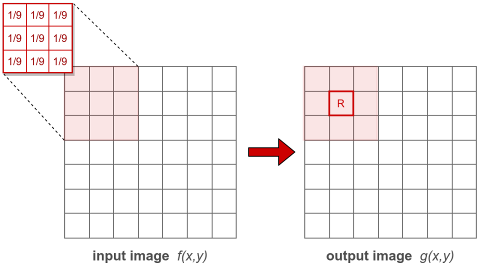

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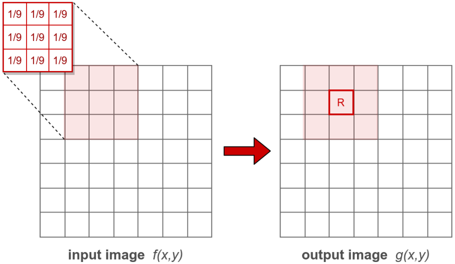

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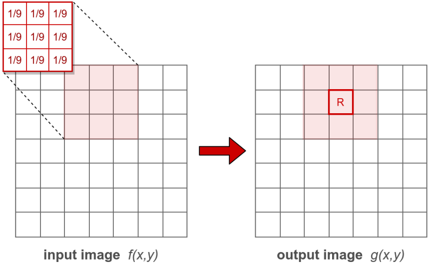

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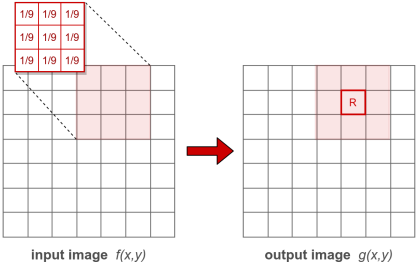

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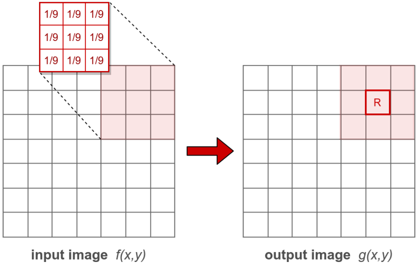

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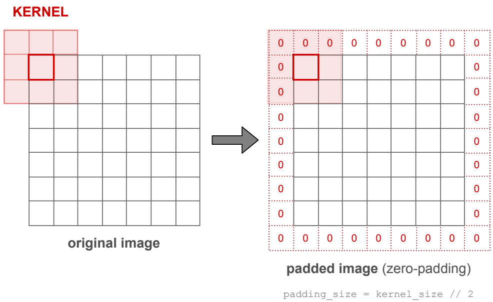

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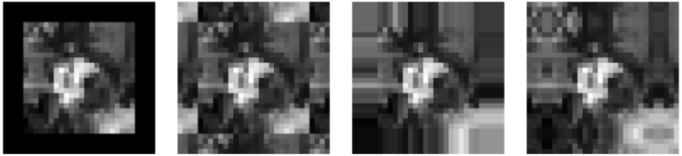
various padding types (Richard Szeliski, 2010)



zero wrap clamp mirror

Linear spatial filter

$\Rightarrow$ the sum-of-products operation between the input image $f(x, y)$ and filter kernel w (eq.1) is the implementation of a **spatial convolution** (eq.2):

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) \cdot f(x - s, y - t) \tag{1}$$

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linear spatial filtering $\Longleftrightarrow$ **spatial convolution**

convolutions are the core operations used by **Convolutional Neural Networks** (CNN)

Kernel coefficients define the nature of the filter
$\Rightarrow$ vary kernels coefficients according to the desired filtering operation:

- **smoothing** filters
  $\Rightarrow$ **low-pass** filters $\rightarrow$ *retains low-frequency components of the image*
  - *averaging* kernel (a.k.a. *box filter*)
  - *gaussian* kernel

- **sharpening** filters
  $\Rightarrow$ **high-pass** filters $\rightarrow$ *retains high-frequency components of the image*
  $\Rightarrow$ **edge detection** filters:
  - *Sobel* kernel, *Prewitt* kernel, etc. $\rightarrow$ directional filters (sensitive to edge orientation)
  - *Laplacian* kernel $\rightarrow$ isotropic filter (not sensitive to edge orientation)

  $\Rightarrow$ **sharpening** filters: increase image contrast along edges

- **other**
  - *Emboss* filter $\rightarrow$ appearance of the image being "embossed" or elevated from the background

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2.3. kernels types and applications

original image

identity

0	0	0
0	1	0
0	0	0

filtered image

?

2.3. kernels types and applications

original image

identity

0	0	0
0	1	0
0	0	0

filtered image

$\Rightarrow$ no change!

2.3. kernels types and applications

original image

average

0.1	0.1	0.1
0.1	0.1	0.1
0.1	0.1	0.1

filtered image

?

LOW-PASS FILTER

original image

average

0.1	0.1	0.1
0.1	0.1	0.1
0.1	0.1	0.1

filtered image

unweighted average, a.k.a. **box filter**

$\Rightarrow$ blurring effect

2.3. kernels types and applications

original image

gaussian

.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.1	.0	.0	.0
.0	.0	.1	.2	.1	.0	.0
.0	.1	.2	.4	.2	.1	.0
.0	.0	.1	.2	.1	.0	.0
.0	.0	.0	.1	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0

filtered image

?

LOW-PASS FILTER

original image

gaussian

.0	.0	.0	.0	.0	.0	.0
.0	.0	.0	.1	.0	.0	.0
.0	.0	.1	.2	.1	.0	.0
.0	.1	.2	.4	.2	.1	.0
.0	.0	.1	.2	.1	.0	.0
.0	.0	.0	.1	.0	.0	.0
.0	.0	.0	.0	.0	.0	.0

filtered image

weighted average

$\Rightarrow$ blurring effect with more weight on central pixel

2.3. kernels types and applications

original image

laplacian

0	-1	0
-1	4	-1
0	-1	0

filtered image

?

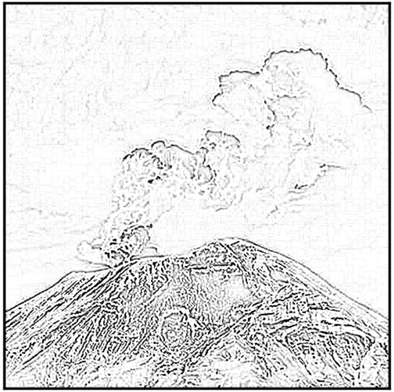
HIGH-PASS FILTER

original image

laplacian

0	-1	0
-1	4	-1
0	-1	0

filtered image



(extension of the Laplacian kernel)

$\Rightarrow$ edge detection (no orientation)

2.3. kernels types and applications

original image

sharpen

0	-1	0
-1	5	-1
0	-1	0

filtered image

?

HIGH-PASS FILTER

original image

sharpen

0	-1	0
-1	5	-1
0	-1	0

filtered image

identity kernel + highpass kernel

$\Rightarrow$ sharpening effect

2.3. kernels types and applications

original image

sobel x

-1	0	1
-2	0	2
-1	0	1

filtered image

?

HIGH-PASS FILTER

original image

sobel x

-1	0	1
-2	0	2
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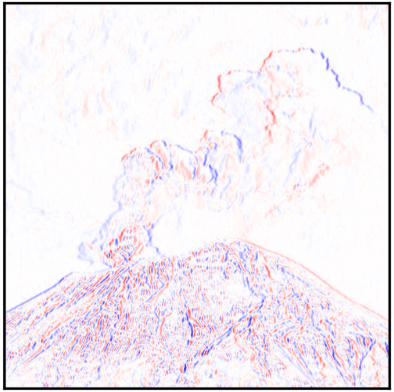
filtered image



$\Rightarrow$ edge detection (x-direction)

2.3. kernels types and applications

original image

sobel y

-1	-2	-1
0	0	0
1	2	1

filtered image

?

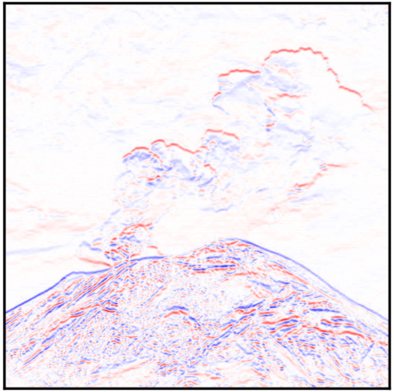
HIGH-PASS FILTER

original image

sobel y

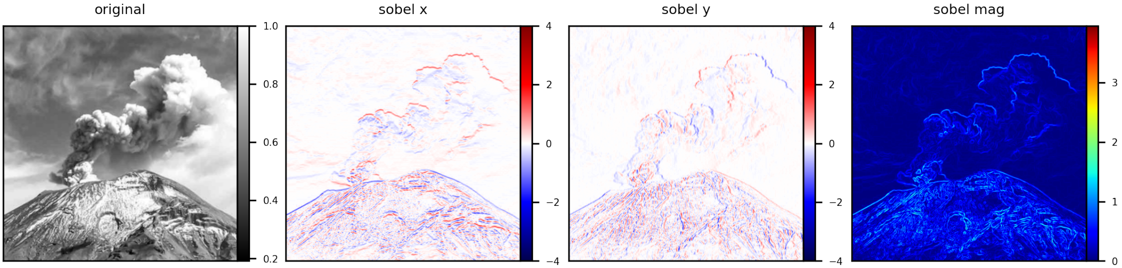
-1	-2	-1
0	0	0
1	2	1

filtered image



$\Rightarrow$ edge detection (y-direction)

2.3. kernels types and applications



$\Rightarrow$ edges + magnitude

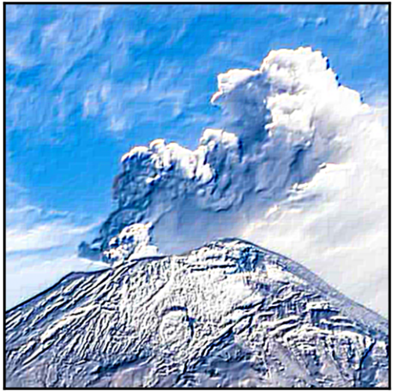
2.3. kernels types and applications

original image

emboss

-2	-1	0
-1	1	1
0	1	2

filtered image

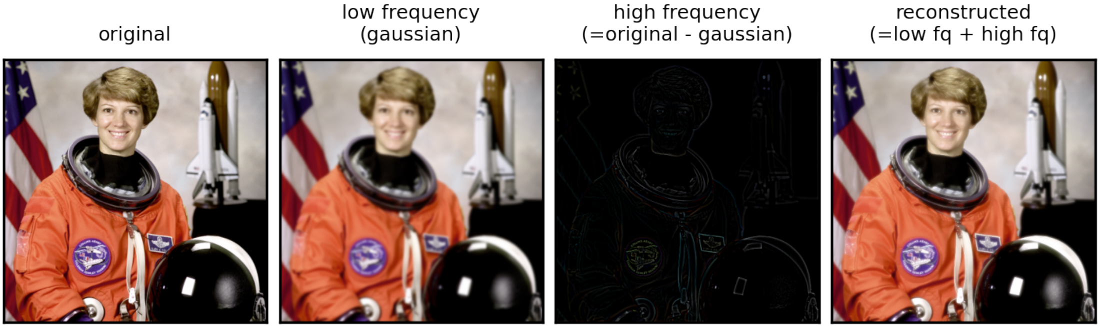


$\Rightarrow$ styling effect

Gaussian filters are a true low-pass filter for the image

$\Rightarrow$ we can retrieve the low-frequency in an image

$\Rightarrow$ we can retrieve the high-frequency in an image by subtracting the low-frequency from the original image



3.1. 1D Fourier transform

⇒ convolutions for **spatial domain filtering** is powerful, BUT it has high computational costs

⇒ **frequency domain filtering** offers computational advantages:

*(**convolution** in the time domain $\Longleftrightarrow$ **multiplication** in the frequency domain)*

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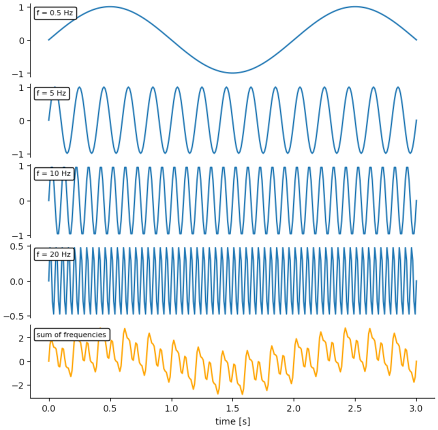
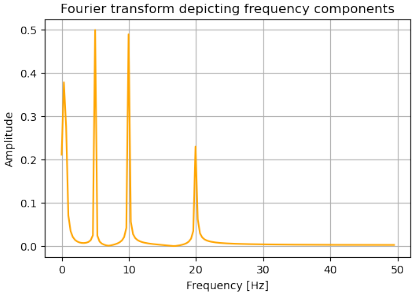
*(**convolution** in the time domain $\Longleftrightarrow$ **multiplication** in the frequency domain)*

3.1. 1D Fourier transform

Fourier theorem: a continuous and periodic function can be approximated as infinite sum of sine- and cosine-functions

- **Forward transform**: Time Domain → Frequency Domain
- **Inverse transform**: Frequency Domain → Time Domain



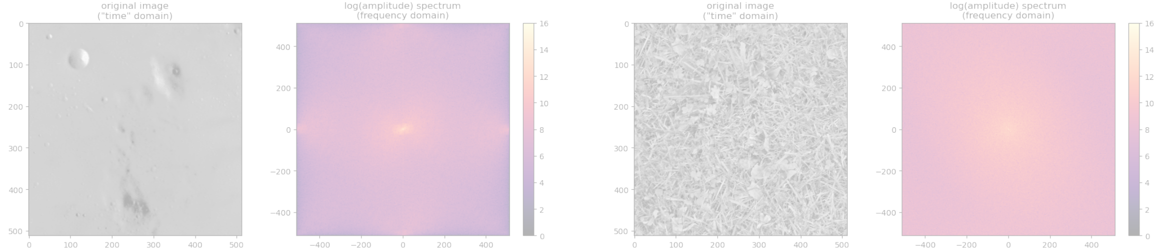


Fourier transform on images ?

⇒ an image can also be expressed as the sum of sinusoids of different frequencies and amplitudes

⇒ the appearance of an image depends on the frequencies of its sinusoidal components:
(NB: Fourier transform of a real function is symmetric about the origin; by convention frequency 0 is set at the center of image)

- **low frequencies** → regions with intensities that vary slowly
- **high frequencies** → edges and other sharp intensity transitions



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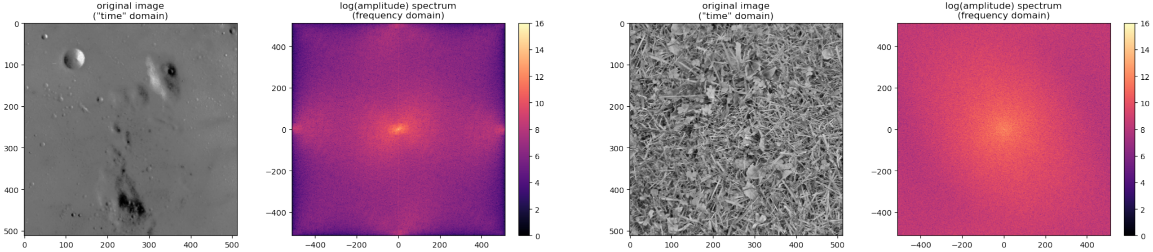
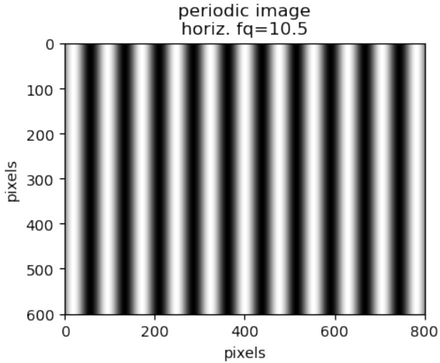
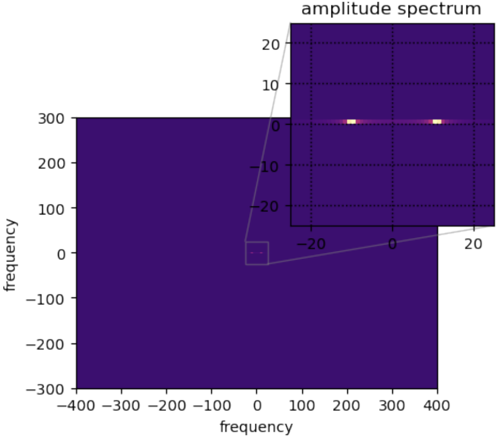


2D Fourier transform on SYNTH images

⇒ "dots" symmetric about origin in amplitude spectrum
⇒ distance/direction from origin imply frequency in time domain





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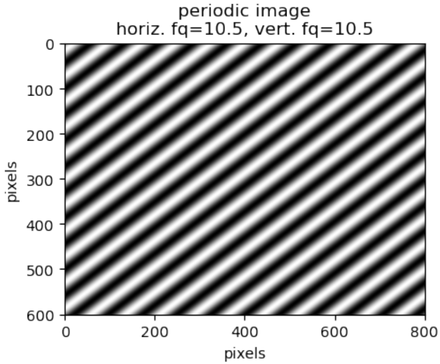

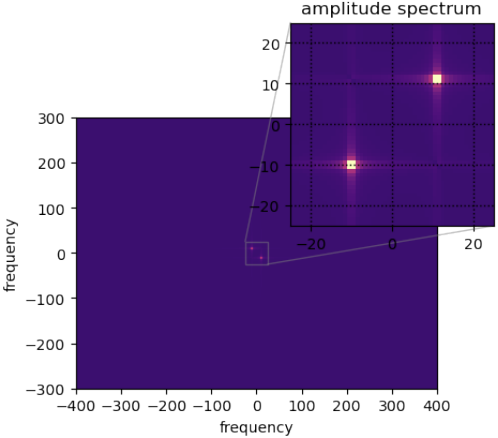

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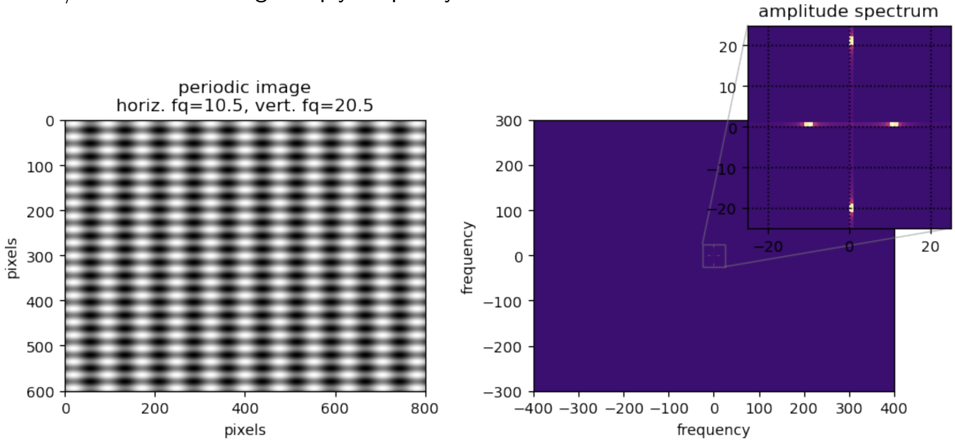
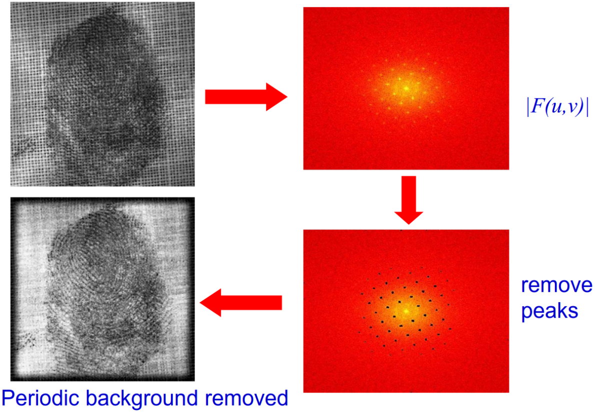
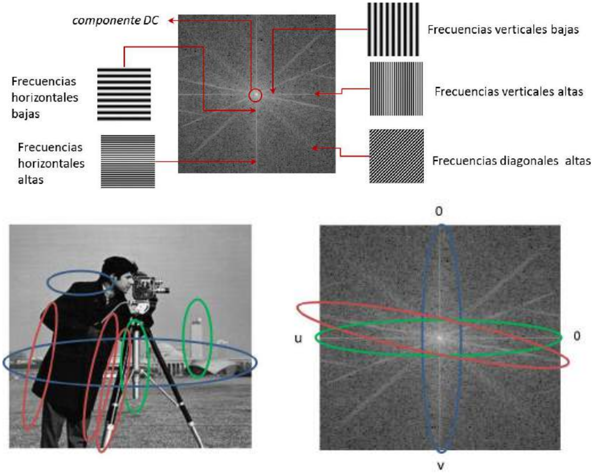


2D Fourier transform on REAL images



Credit: A. Zisserman

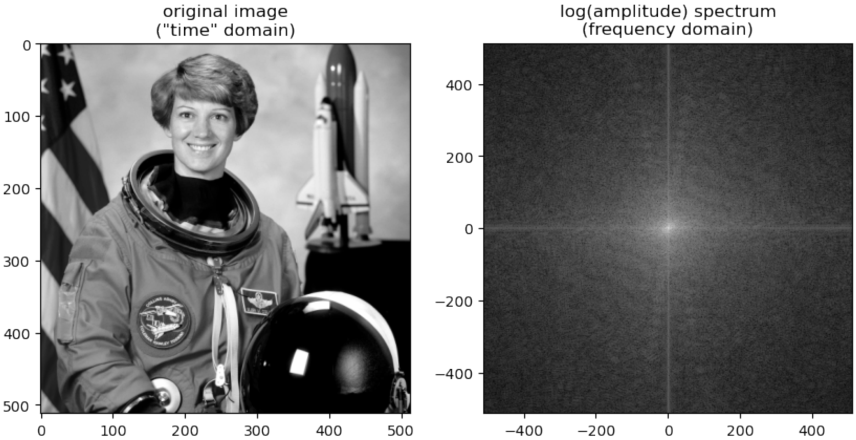
2D Fourier transform on REAL images



Credit: Alegre et al. 2016

2D Fourier transform on REAL images

⇒ let's try on our astronaut



2D Fourier transform on REAL images

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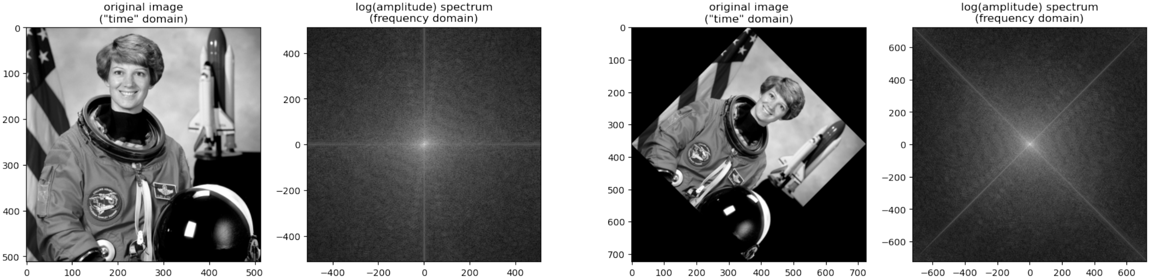
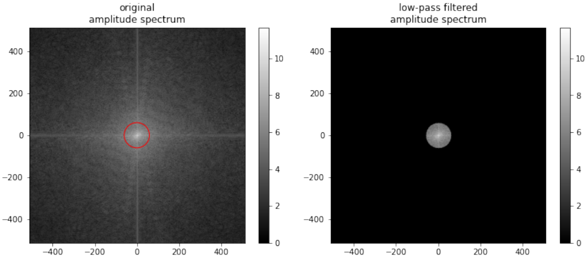


2D Fourier transform on REAL images

$\Rightarrow$ band-pass image frequencies?

- **low-pass** filter $\rightarrow$ cut off high-frequencies
- **high-pass** filter $\rightarrow$ cut off low-frequencies



2D Fourier transform on REAL images

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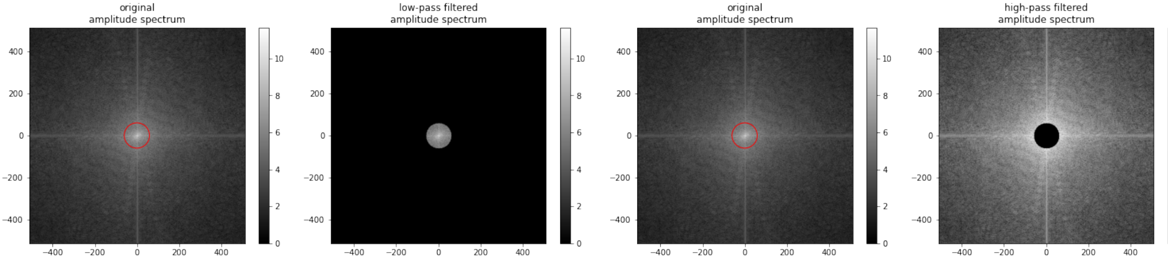
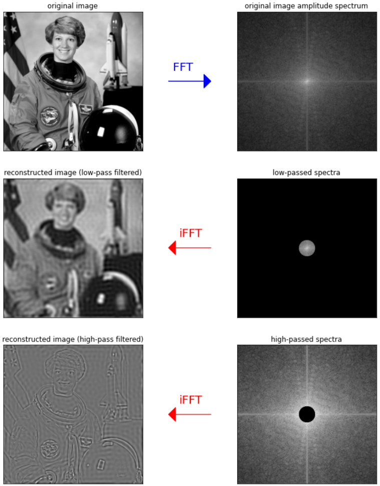


2D Fourier transform on REAL images

⇒ image can be reconstructed from band-passed spectra using the 2D **inverse Fourier transform** (iFFT2)



2D Fourier transform on REAL images

$\Rightarrow$ the **ideal low-pass filter** (LPF) introduces artefacts:
- "ripples" near strong edges in the original image: **ringing effect**
- related to the sharp cut-off in ideal frequency domain

low-pass filtered image

ringing effect

2D Fourier transform on REAL images

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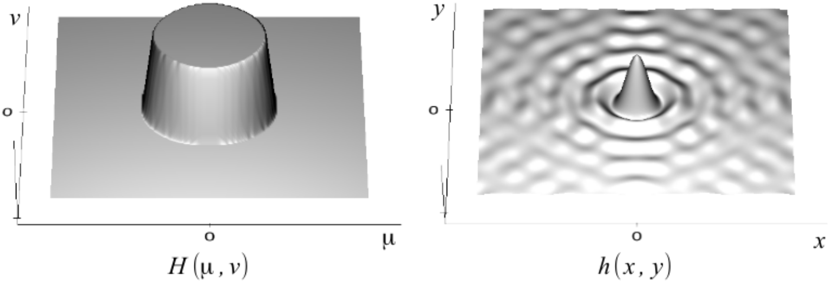
- related to the sharp cut-off in ideal frequency domain

frequency domain

time domain

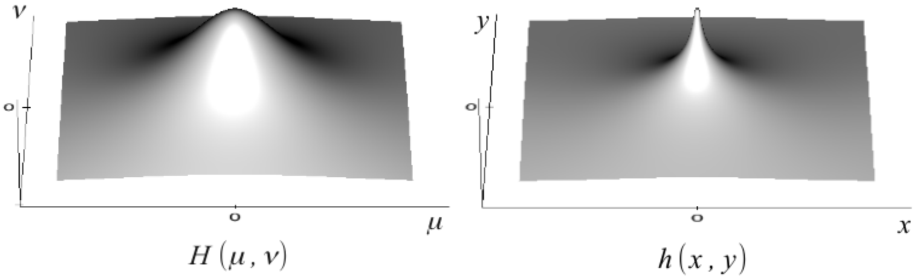


- Ideal LPF has significant 'side-lobes' in the time domain

2D Fourier transform on REAL images

⇒ the **Butterworth** filter offers impulse response without side-lobes in the time domain ideal
→ no "ringing effect", due to the absence of discontinuity in spectrum

frequency domain time domain



- Impulse response without side-lobes in the time domain

2D Fourier transform on REAL images

$\Rightarrow$ the **Butterworth** filter offers impulse response without side-lobes in the time domain ideal
$\rightarrow$ no "ringing effect", due to the absence of discontinuity in spectrum