Machine Learning 2/3

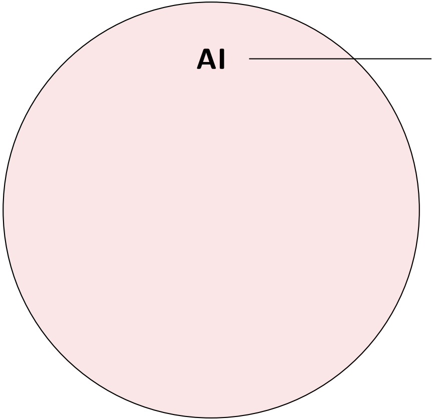
Lecture 08

Computer Vision for Geosciences

2021-04-30

UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO

Pinpoint “hot” words



Artificial Intelligence

broad concept, whereby machine mimics human behaviour

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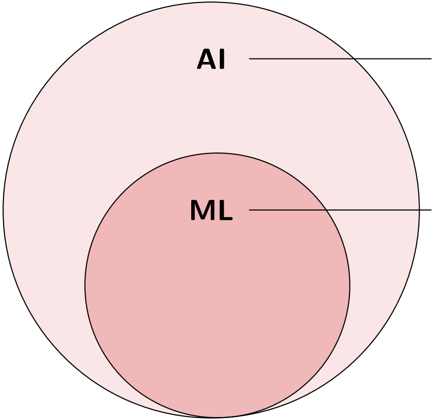

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(features are designed by the user)

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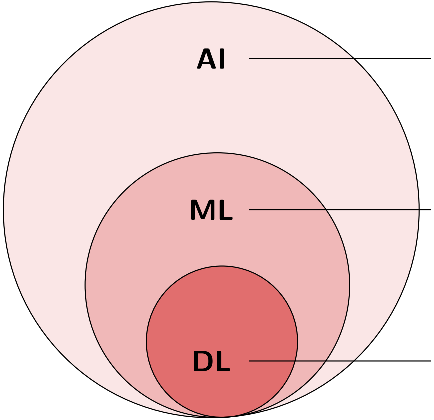

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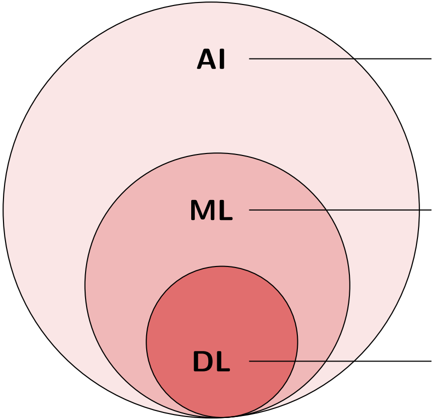

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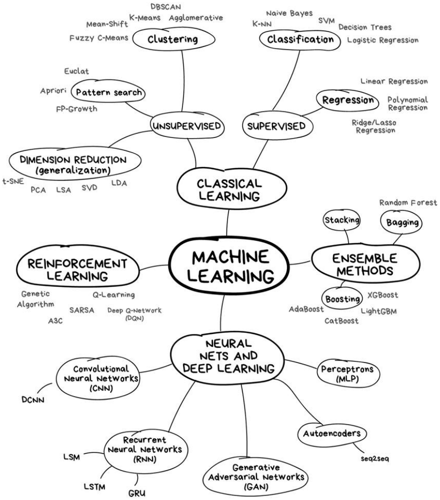
ML: lectures 07, 08 (today), 09
DL: lectures 10, 11, 12

Machine Learning is a huge (and growing) field!



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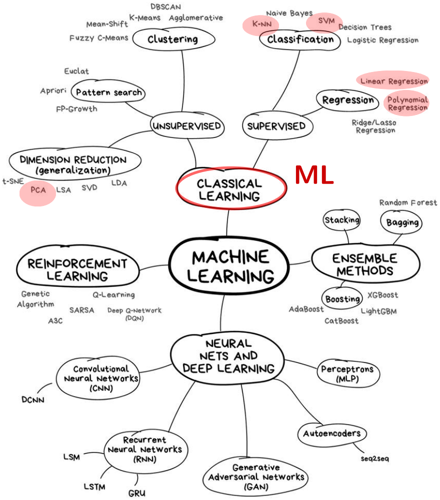

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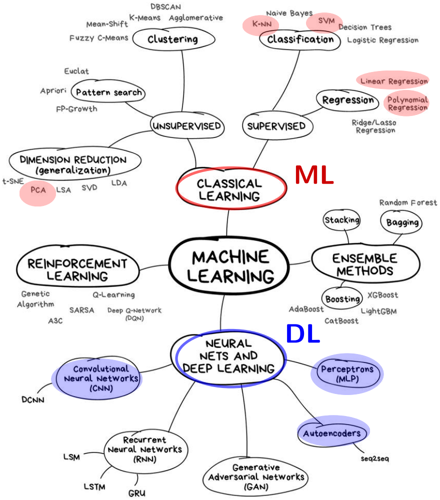

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What we will introduce in the ML lectures:

Machine Learning

Supervised Learning

- Learning algorithm is presented inputs and desired outputs: **training data** $D = (in, out)$
- Goal: learn a general rule f that maps inputs to outputs $f(in) = out$

⇒ **Regression task**: *out* is a *continuous* number
e.g. linear regression, polynomial regression

⇒ **Classification task**: *out* is a *nominal* number (class label)
e.g. kNN, SVM, Logistic Regression

Unsupervised Learning

- No training data is given to the learning algorithm
- Goal: find structure data, discover hidden patterns, learn features

⇒ **Dimension reduction**, e.g. PCA
→ also used to craft features

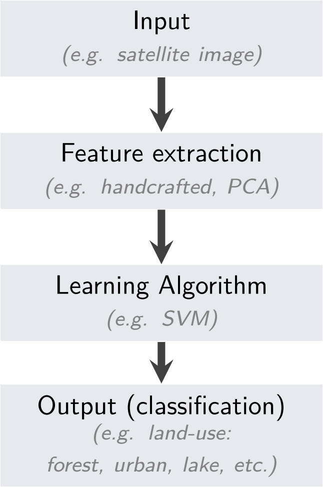
⇒ **Clustering task**, e.g. K-means

Classification task

- Goal:
  Learn the mapping between low level **features**, and **high level information** *(e.g. semantic classes)*

  NB: "features" is here used in a broad sense, not the "descriptors" introduced in lecture 06 (e.g. HOG, SIFT)

- Steps:
  1. features extraction *(e.g. handcrafted, PCA)*
  2. learning algorithm *(e.g. SVM)*



Toy example (courtesy of Andreas Ley & Ronny Hänsch)

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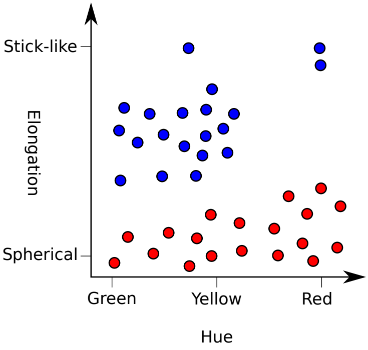

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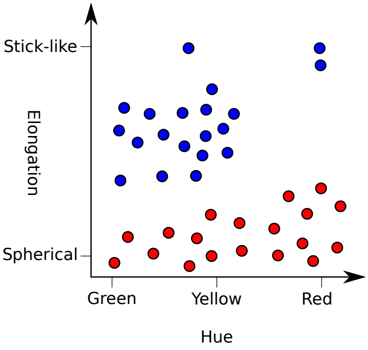

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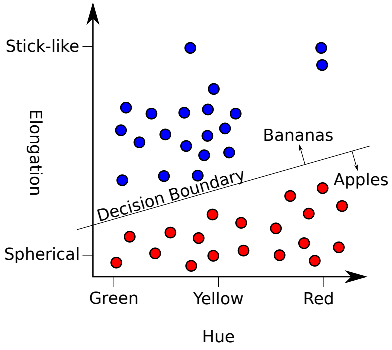

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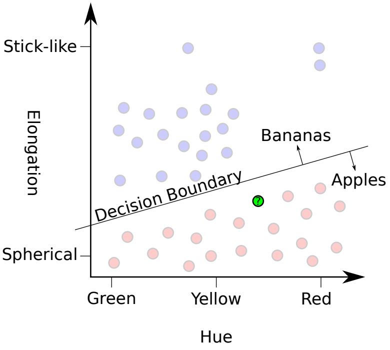

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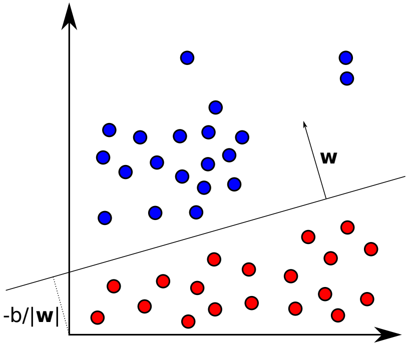
  $\Rightarrow$ **perceptron**: $\boxed{y = \text{sign}(\mathbf{w}^T\mathbf{x} + b)}$
  - $y \in \{-1, 1\}$: predicted class $\rightarrow$ *banana or apple*
  - $\mathbf{x} \in \mathbb{R}^2$: feature vector $\rightarrow$ *[hue, elongation]*
  - $\mathbf{w} \in \mathbb{R}^2$: "weight vector" $\rightarrow$ *needs to be learned*
  - $b \in \mathbb{R}$: "bias" $\rightarrow$ *needs to be learned*
  - *sign*: sign function returning the sign of a real number



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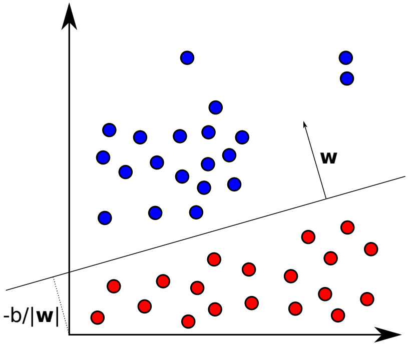
NB: **Support Vector Machine (SVM)**
can be used to find the best decision boundary
(i.e. which maximizes distance to data points)
$\rightarrow$ **next lecture!**



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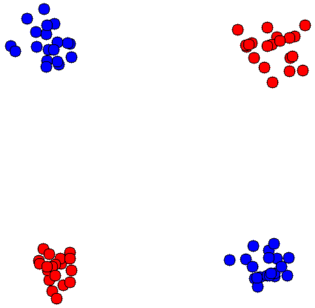
- **Problem**:
  ⇒ feature space often not linearly separable



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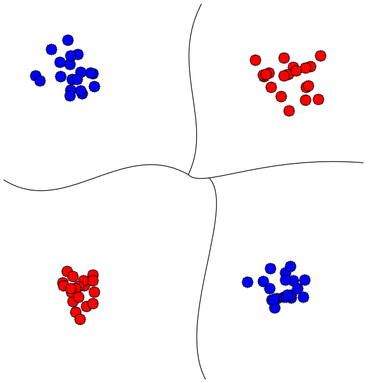

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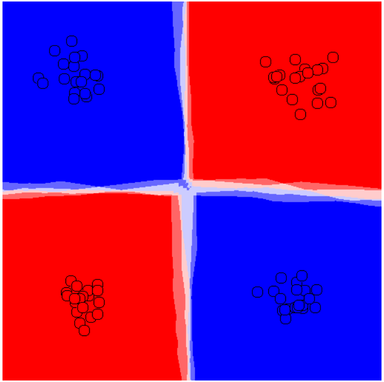

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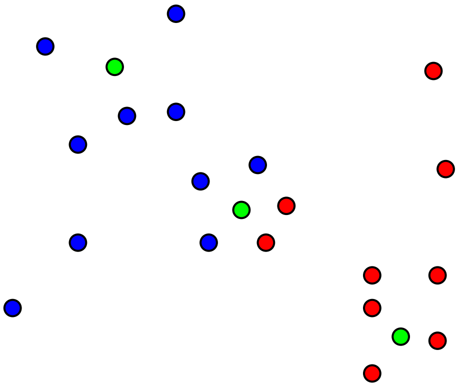

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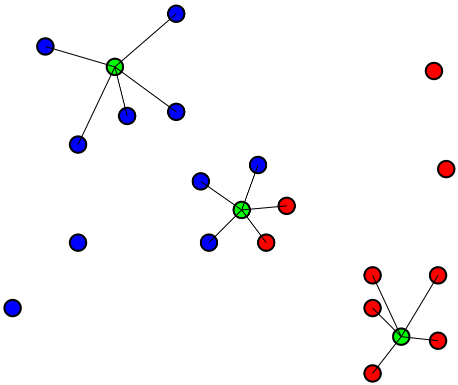

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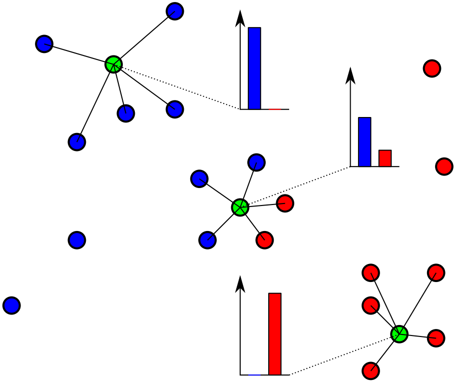

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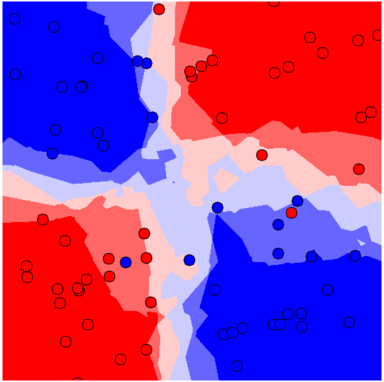
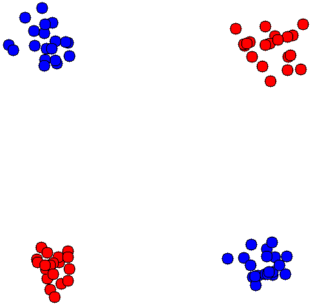
  4. decision boundary can be designed as probability meshgrids

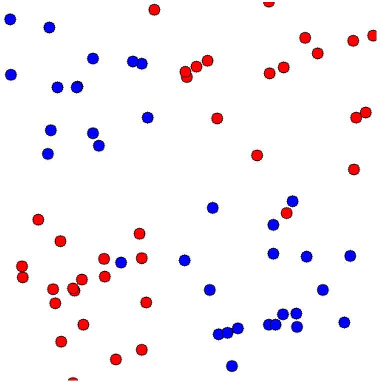


kNN examples

simple case



hard case



kNN examples

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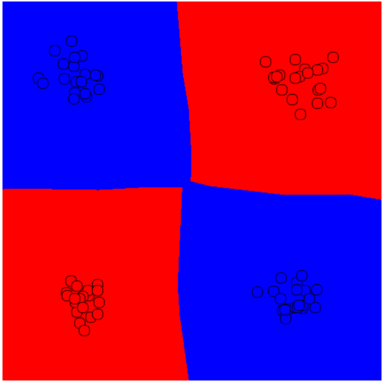

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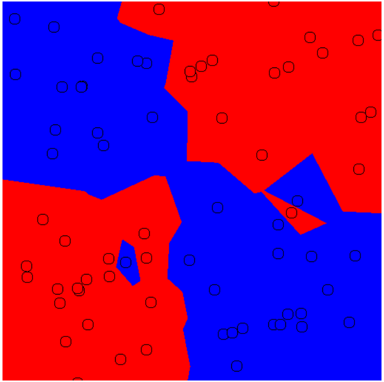

kNN examples

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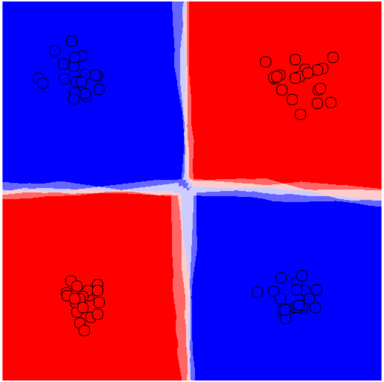

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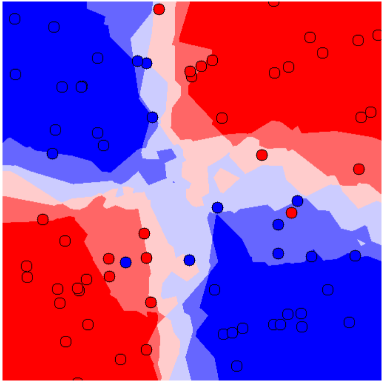

kNN examples

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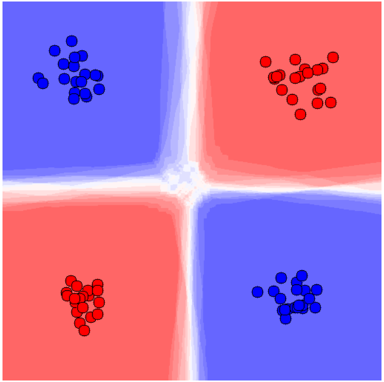

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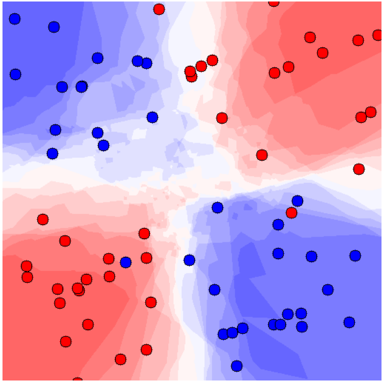

Feature extraction:

⇒ handcrafting features is nice, but can we do better?

Input

(e.g. satellite image)

↓

Feature extraction

*(e.g. handcrafted, **PCA**)*

↓

Learning Algorithm

(e.g. SVM)

↓

Output (classification)

(e.g. land-use)

Feature extraction:

⇒ handcrafting features is nice, but can we do better?

⇒ find a space where samples from different classes are well separable

Input *(e.g. satellite image)*
↓
Feature extraction *(e.g. handcrafted, **PCA**)*
↓
Learning Algorithm *(e.g. SVM)*
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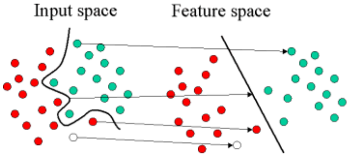

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EX: intuitive representation of PCA (video):

How to take a picture to capture the most information about the teapot?

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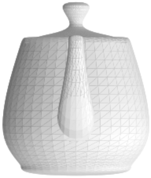

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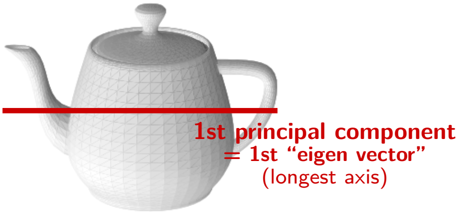

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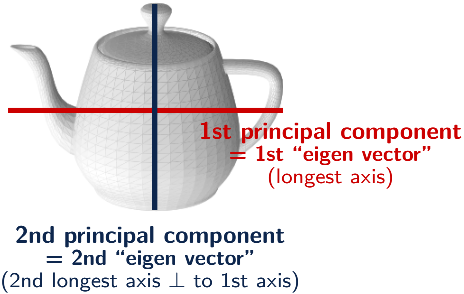

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$\Rightarrow$ **Principal Component Analysis** (PCA) $\rightarrow$ *represent data in a space that best describes the data variation*

$\Rightarrow$ PCA can be used to reduce data dimensions $\rightarrow$ *will reduce computational load of the classifier*

PCA toy example (inspired by this post)

*We have several wine bottles in our cellar, described by different **features**: alcohol, color, etc. However many features will measure related properties, and so will be redundant.*

	alcohol	malic_acid	ash	alcalinity_of_ash	magnesium	total_phenols	flavanoids	nonflavanoid_phenols	proanthocyanins	color_intensity	hue
0	14.23	1.71	2.43	15.6	127.0	2.80	3.06	0.28	2.29	5.64	1.04
1	13.20	1.78	2.14	11.2	100.0	2.65	2.76	0.26	1.28	4.38	1.05
2	13.16	2.36	2.67	18.6	101.0	2.80	3.24	0.30	2.81	5.68	1.03
3	14.37	1.95	2.50	16.8	113.0	3.85	3.49	0.24	2.18	7.80	0.86
4	13.24	2.59	2.87	21.0	118.0	2.80	2.69	0.39	1.82	4.32	1.04
...	...	...	...	...	...	...	...	...	...	...	...

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$\Rightarrow$ **reduce data dimensions**

$\Rightarrow$ PCA does *not* select some features and discards others,
instead it **defines new features** (using linear combinations of available features)
which will best represent wine variability

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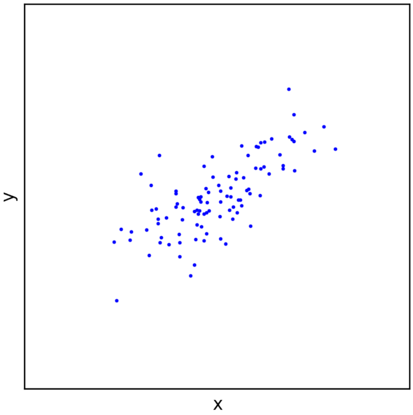
1	13.20	1.78	2.14	11.2	100.0	2.65	2.76	0.26	1.28	4.38	1.05
2	13.16	2.36	2.67	18.6	101.0	2.80	3.24	0.30	2.81	5.68	1.03
3	14.37	1.95	2.50	16.8	113.0	3.85	3.49	0.24	2.18	7.80	0.86
4	13.24	2.59	2.87	21.0	118.0	2.80	2.69	0.39	1.82	4.32	1.04
...	...	...	...	...	...	...	...	...	...	...	...

⇒ PCA allows to summarize each wine with fewer characteristics
⇒ **reduce data dimensions**

⇒ PCA does *not* select some features and discards others,
instead it **defines new features** (using linear combinations of available features)
which will best represent wine variability

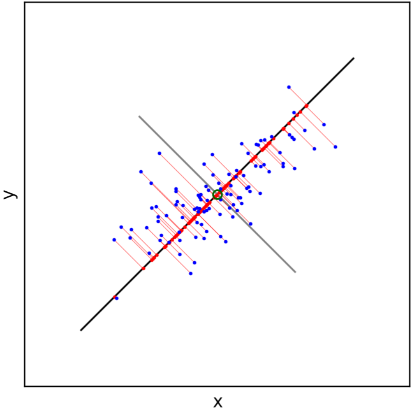
How?

Consider 2 correlated features x and y:



Consider 2 correlated features x and y:

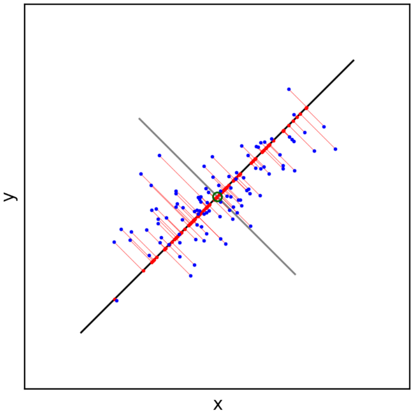
$\Rightarrow$ a **new "feature"** (red dots •) can be constructed by drawing a line through the cloud and projecting all points onto it



Consider 2 correlated features x and y:

$\Rightarrow$ a **new "feature"** (red dots •) can be constructed by drawing a line through the cloud and projecting all points onto it

$\Rightarrow$ **linear combination** $w_1 x + w_2 y$



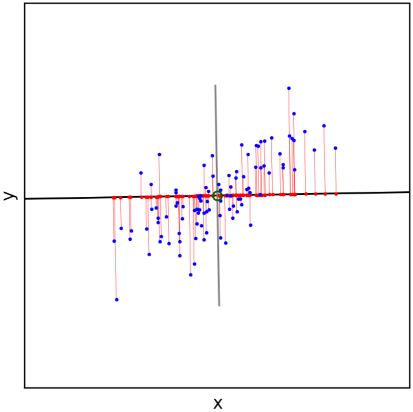
Consider 2 correlated features x and y:

⇒ a **new "feature"** (red dots •) can be constructed by drawing a line through the cloud and projecting all points onto it

⇒ **linear combination** $w_1 x + w_2 y$

⇒ PCA will find the "best" line according to 2 criteria:

- maximum **variance** of the red dots (i.e., spread along black line)
- minimum **distance** to black line (i.e., length of red lines)



NB: the above animation will only run with PDF readers having built-in JavaScript engine (ex: Adobe Reader, recent versions of Okular, etc.)

Consider 2 correlated features x and y:

⇒ a **new "feature"** (red dots •) can be constructed by drawing a line through the cloud and projecting all points onto it

⇒ **linear combination** $w_1x + w_2y$

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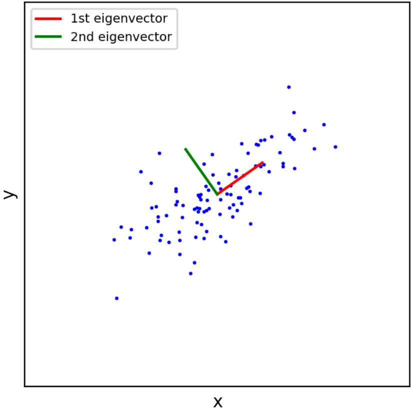
⇒ "best" line = **1st eigenvector** = **1st principal component**



Consider 2 correlated features x and y:

⇒ a **new "feature"** (red dots ●) can be constructed by drawing a line through the cloud and projecting all points onto it

⇒ **linear combination** $w_1 x + w_2 y$

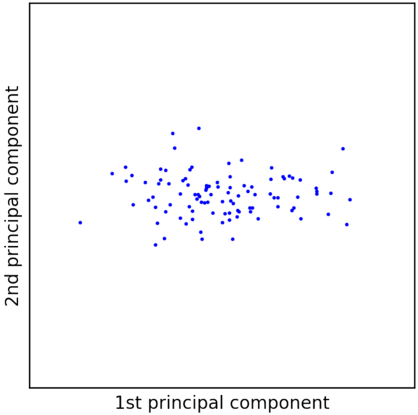
⇒ PCA will find the "best" line according to 2 criteria:

- maximum **variance** of the red dots (i.e., spread along black line)
- minimum **distance** to black line (i.e., length of red lines)

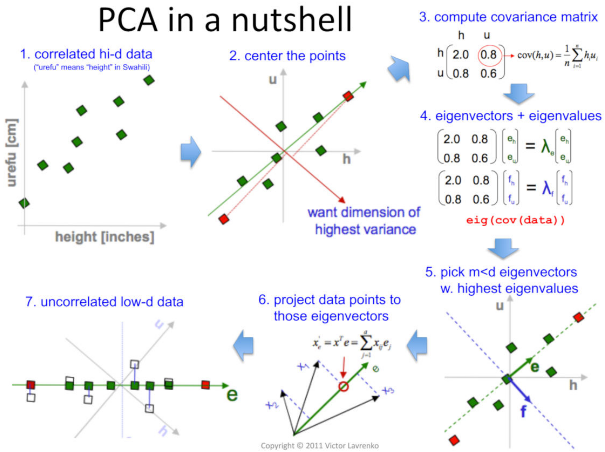
⇒ "best" line = **1st eigenvector** = **1st principal component**

⇒ we can project the data on the principal components, and thereby reduce dimensionality

NB: if only one eigenvector was kept, the transformed data would have only one dimension



⇒ **PCA implementation steps** (video link):



EXERCICE:

PCA analysis on satellite image crops

Math reminders

$$
\begin{aligned}
\textbf{variance } \sigma^2 &= \text{measure of the ``spread'' or ``extent'' of the data about some particular axis} \\
&= \text{average of the squared differences from the mean} \\
&= \text{square of standard deviation } (\sigma) \\
var_x &= \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N} \\
var_y &= \frac{\sum_{i=1}^{N}(y_i - \bar{y})^2}{N}
\end{aligned}
$$

$$
\begin{aligned}
\textbf{covariance} &= \text{measure the level to which two variables vary together.'' of the joint variability of two random variables} \\
cov_{x,y} &= \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{N-1} \\
\textbf{covariance matrix} &= \begin{bmatrix} var_x & cov_{x,y} \\ cov_{x,y} & var_y \end{bmatrix}
\end{aligned}
$$